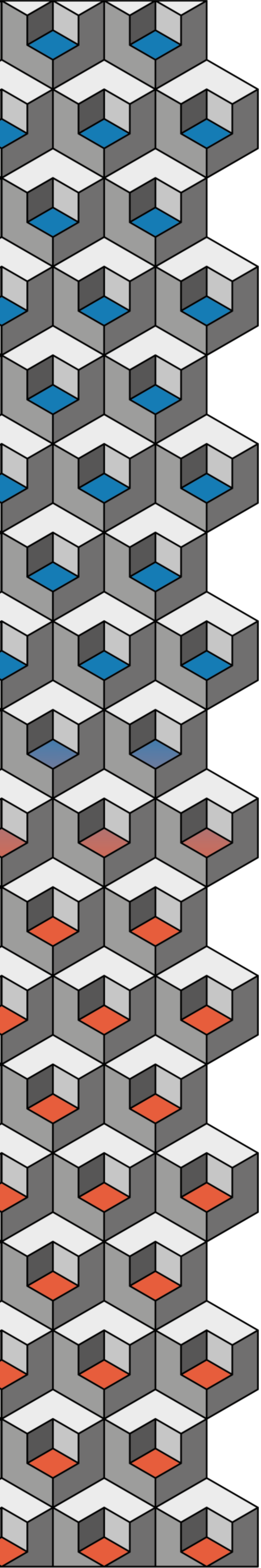

GEOMETRY, GRAPHICS AND DESIGN IN THE DIGITAL AGE

The 9th International Scientific Conference on Geometry and Graphics

MONGEOMETRIJA 2023

Editors
Ivana Bajšanski
Marko Jovanović

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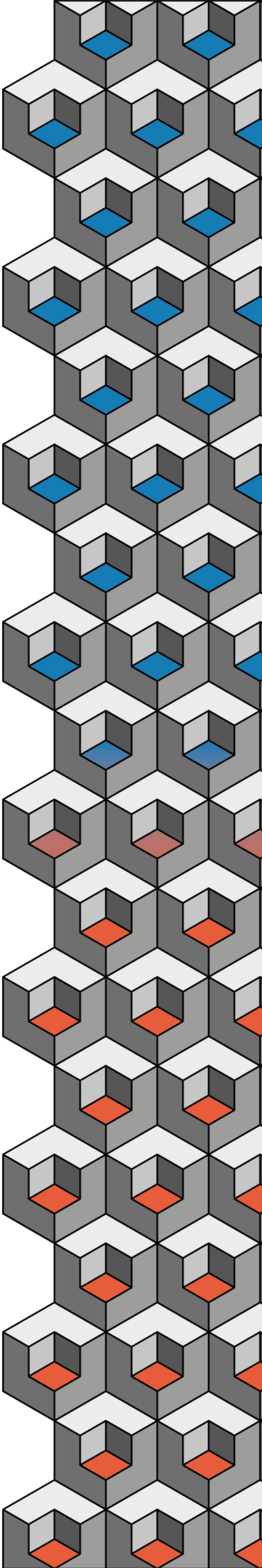
Novi Sad, May 2023

Ivana Bajšanski and Marko Jovanović

MONG2023 Conference chairs



GEOMETRY, GRAPHICS AND DESIGN IN THE DIGITAL AGE

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Table of Contents

TRANSFORMATION OF PENCILS OF CIRCLES INTO PENCILS OF CONICS AND THESE INTO PENCILS OF HIGHER-ORDER CURVES

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Abstract

In this paper, an elliptical pencil of circles is mapped by homology (perspective collineation) into a parabolic-parabolic pencil of hyperbolas because it has two by two common points 1 = 2 and 3 = 4 in infinity, i.e., four overlapping points in the antipode. The mapping also includes the mapping of degenerated conics decomposed into corresponding pairs of straight lines. The elliptical pencil of circles (the ellipses on plane 1) is mapped into a hyperbolic pencil (on plane 2) by perspective collineation for pole S_1. The vanishing line v_1 intersects all the circles of the pencil so that by homology all the circles are mapped into hyperbolas. To apply supersymmetry to the obtained pencils, the angle between plane 1 and plane 2 must be 45°. The pencil of hyperbolas is mapped by supersymmetry into a pencil of higher-order curves. The obtained pencils are further mapped by inversion and then again by supersymmetry to obtain higher-order curves rich in different shapes.

In the second example, the elliptical pencil of circles is placed to the vanishing line v_1 so that by homology, it is mapped into a pencil of conics containing an ellipse (circle 1 does not intersect the vanishing line), a parabola (circle 2 touches the vanishing line), a hyperbola (circle 3 intersects the vanishing line). The pencil of conics is elliptical-parabolic because it has two real and separate points and two infinite, i.e., two overlapping points in the antipode. This pencil of conic is also mapped by supersymmetry into a pencil of higher-order curves that intersect at the same number and type of base points as the starting pencil. Higher order curves obtained by mapping are rational line curves.

Keywords: pencils of circles, pencils of conics, inversion, supersymmetry, pencils of curves of the higher orders

1 INTRODUCTION

Professor Lazar Dovnikovic has made a significant contribution to the study of curves. Modern geometry has introduced the term observer (hence the term relativistic geometry) [3]. The term "plane" is replaced by "sphere" and the term "straight line" by "circle passing through the observer’s antipodal point". Perception and interpretation of geometric elements depend solely on the observer’s position. Through the observer’s standing point on the “plane”, a pencil of “straight” geodesics cuts through. Each of these largest circles on the sphere defines a pencil of straight lines that are parallel to them but are not geodesics. By moving away from the geodesics on both sides, the diameter of the circles decreases. (Fig. 1). For each direction at the antipode, there is one infinitesimal circle. Thus, we come to an unusual conclusion: Unlike the projective plane which has only one infinitely distant straight line for all observers, the relativistic “plane” has infinitely many infinitesimal antipodal “straight lines” for each (of ∞^2) observer individually. However strange it may be, this fact paves the way for creating a simple mechanism for the tying and untying singular points of curves.

A simple example in Fig. 1 shows the essence of relativistic geometry. It explains the concept of parallel lines. Regarding two parallel lines, projective geometry introduces the concept of an infinitely distant point where these two parallel lines intersect. Since parallel lines are equidistant, they can never meet, not even at infinity. Relativistic geometry explains the concept of parallel lines by the fact that an observer standing on a large sphere, on the geodesic marked with d (Fig. 1,d coloured black), perceives all the circles that pass through his antipodal point as parallel or intersecting lines. In the given example, the "straight lines" are parallel to the "d" geodesic of the sphere. By stereographic projection of circles from the antipodal point S_A (red and green circles shown in Fig. 1) onto plane T, a pencil of parallel lines (blue lines) is obtained. Therefore, the observer perceives the red and green circles on his large sphere as blue parallel lines.

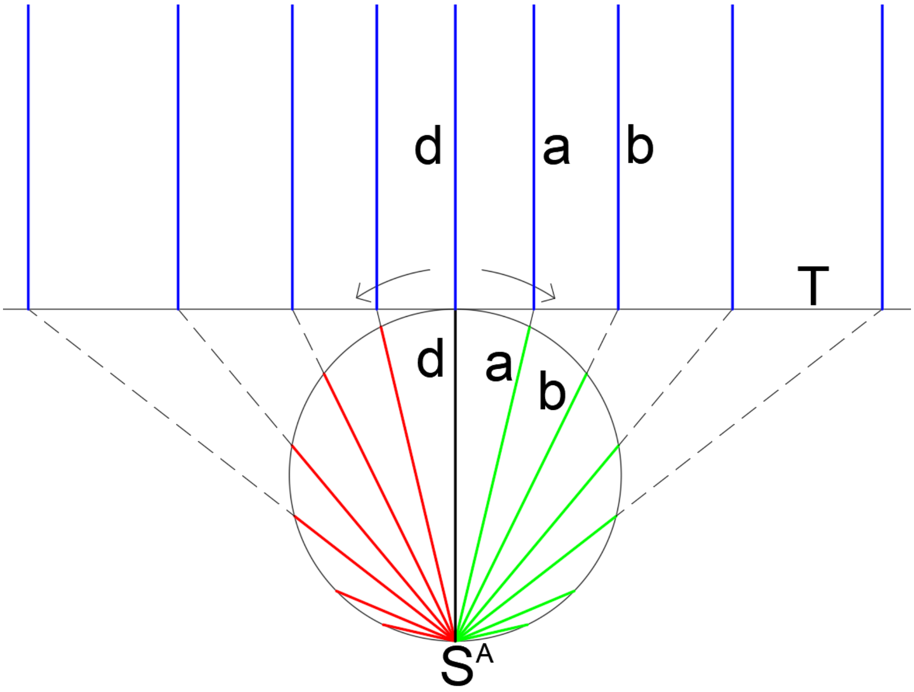

Fig. 1. Parallel lines in relativistic geometry

In relativistic geometry, curves are classified into harmonic groups. Fig. 2 shows the regrouping of curves according to the classical and relativistic order. Figure 1 was taken from "Quantum-relativistic geometry as a new scientific paradigm" [8] .

The relativistic order of the curve equals the sum of its classical order and the number of times the curve passes through the antipode.

The “straight line” and circle are second-order curves because a “straight line” is a circle passing through the observer’s antipodal point. Classical second-order curves have a double point at the antipode, which makes them fourth-order curves in relativistic geometry. The curves of the fourth order of the elliptical type (Fig. 2) have a double isolated point at the antipode. Hyperbolic curves have a double self-intersecting point at the antipode, and parabolic curves have a cusp where the two branches of the curve touch. When the laws of symmetry are applied to curves in a certain group, the properties of higher-order curves will depend on the leading curve in that group. The classification of curves into harmonic groups is of great value in the theory of geometry [4].

2 TRANSFORMATION OF AN ELLIPTICAL PENCIL OF CIRCLES

The following segment shows the mapping of elliptic pencils of circles into corresponding pencils of conics by homology (perspective collineation) which are then mapped by symmetry (for the centre S and the circular axis s) into pencils of higher-order curves. An inversion is then applied to the obtained curves (for the centre S_i and the circular axis s_i) [2] .

2.1 Parabolic-Parabolic (PP) pencil of conics

The elliptical pencil of circles (i.e. ellipses on plane 1 seen as circles in the first projection) which is projected into (2) elliptical pencil of hyperbolas with two pencils of parallel asymptotes is shown in Fig.3.

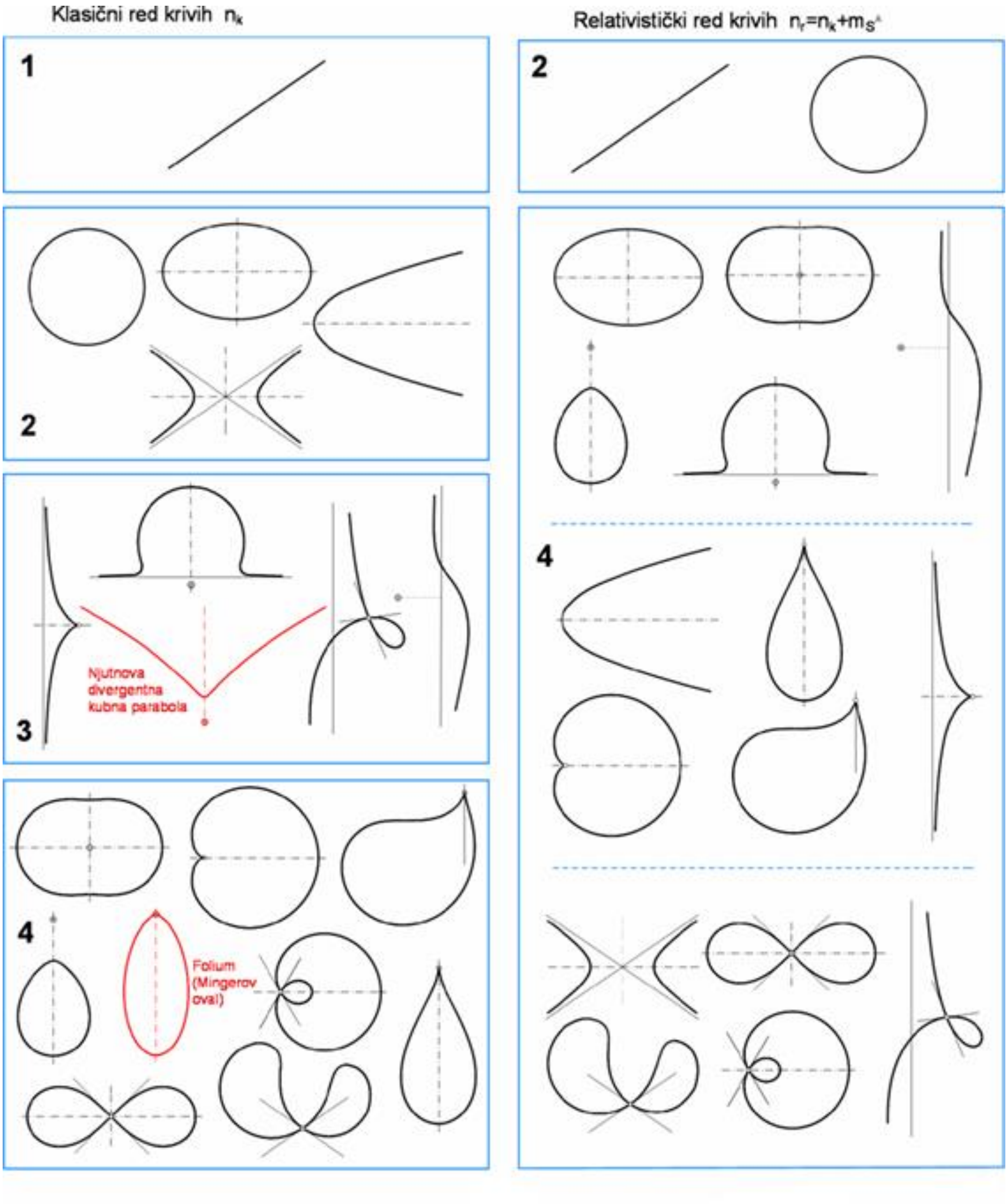

Fig. 2. Classical and relativistic order of curves

Three circles, marked 1, 2 and 3, are mapped into hyperbolas h_1, h_2 and h_3. Vanishing line v_1 intersects all the circles of the pencil, so all circles are mapped into hyperbolas using perspective collineation.The pencil of hyperbolas is parabolic-parabolic (homothetic hyperbolas) because it has two pairs of common points 1=2 and 3=4 at infinity, i.e. four points overlapping at the antipodal point. The disintegrated conic of the pencil is represented by two straight lines that intersect in S (SA and SB). Like mapping rays that pass through S, they are mapped into themselves. The mapped points $\overline{A}$ and $\overline{B}$ are at the observer's antipode. The asymptotes of all hyperbolas are parallel to these rays. A

Lisp routine was written to draw the hyperbola [7]. For that purpose, the UCS (user coordinate system) is inserted into the centre of the hyperbola. The parameters a and b of the hyperbola are loaded, with the condition that the minimum value along the axis x must be a, and the maximum value along the axis x is set at 150 mm (or more). A step of 0.5 mm is set to obtain the required quality of curve smoothness. To follow the mapping of hyperbolas, they are coloured with three different colours.

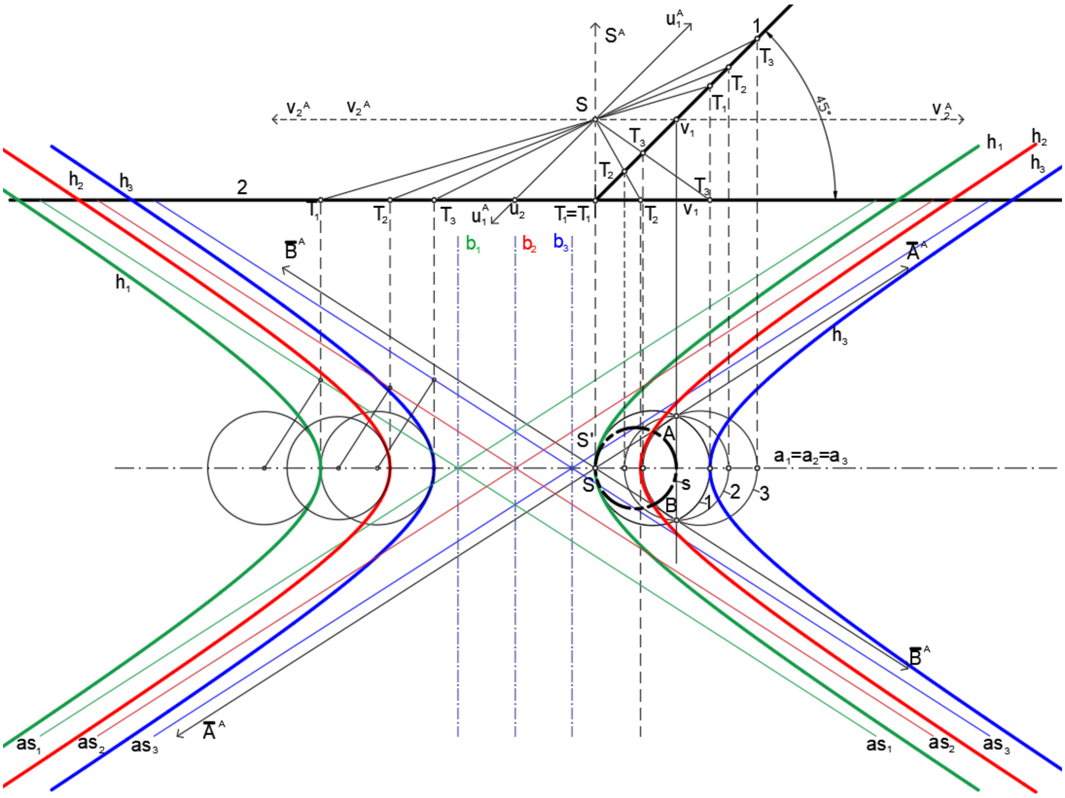

Fig. 3. Parabolic-parabolic pencil of hyperbolas

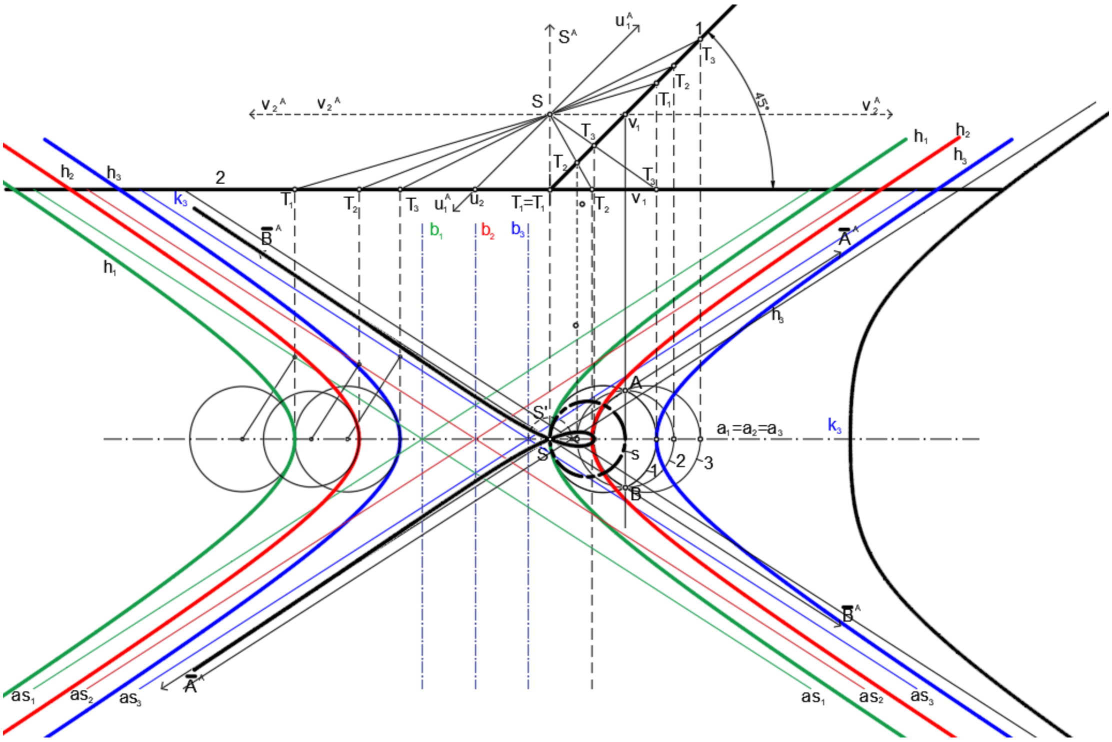

Fig. 4. Mapping of hyperbola numbered 3 into a curve of the eighth order (S,s)

A Lisp program was written for drawing the mapped curves using nonconformal symmetry. First, the hyperbola numbered three was mapped into the curve of the eighth order (S,s) [6]. The program was run by placing the UCS at the centre of symmetry S, loading the radius of the absolute and simply clicking on the curve to be mapped. To write the program for symmetry mapping, we created mathematical equations for the coordinates of the mapped points using the UCS coordinate system in centre S [10]. The following equations for the coordinates of the mapped points $\overline{P}(\overline{x},\overline{y})$ were used provided that the mapped point is P(x,y) [5]:

$$\overline{x}=\frac{4rx^2}{x^2+y^2}-x \qquad \overline{y}=\frac{4rxy}{x^2+y^2}-y$$

Applying axial symmetry, the mapping of points is performed simply by shifting the distance between point P and circular axis s to the other side in the direction of the centre of symmetry S.

Fig. 5 shows simple axial symmetry obtained by mapping points named M_1 and N_1. The points are in plane 1 and are mapped by perspective collineation through the centre S into plane 2 (points M_2 and N_2). Points $M_{1'}$ and $M_{2'}$ are then at the base point (for centre S and inversion circle s_S) inverted into points M_1 and M_2. Points N_1' and N_2' are also inverted into points N_1 and N_2. The points on the absolute which are used to shift the distance of the points from the absolute to the other side of the centre S are marked with the red letter C for M_1 and M_2 and with the blue letter C for N_1 and N_2. The large blue semicircle shows the obtaining of point M_2 by mapping symmetry mapping from point M_1. The smaller blue semicircle shows obtaining of point N_2 by mapping point N_1. The radius of the absolute - s in axial symmetry is equal to half the radius of the circle of inversion - s_S, and the centres of S for inversion and symmetry coincide [9].

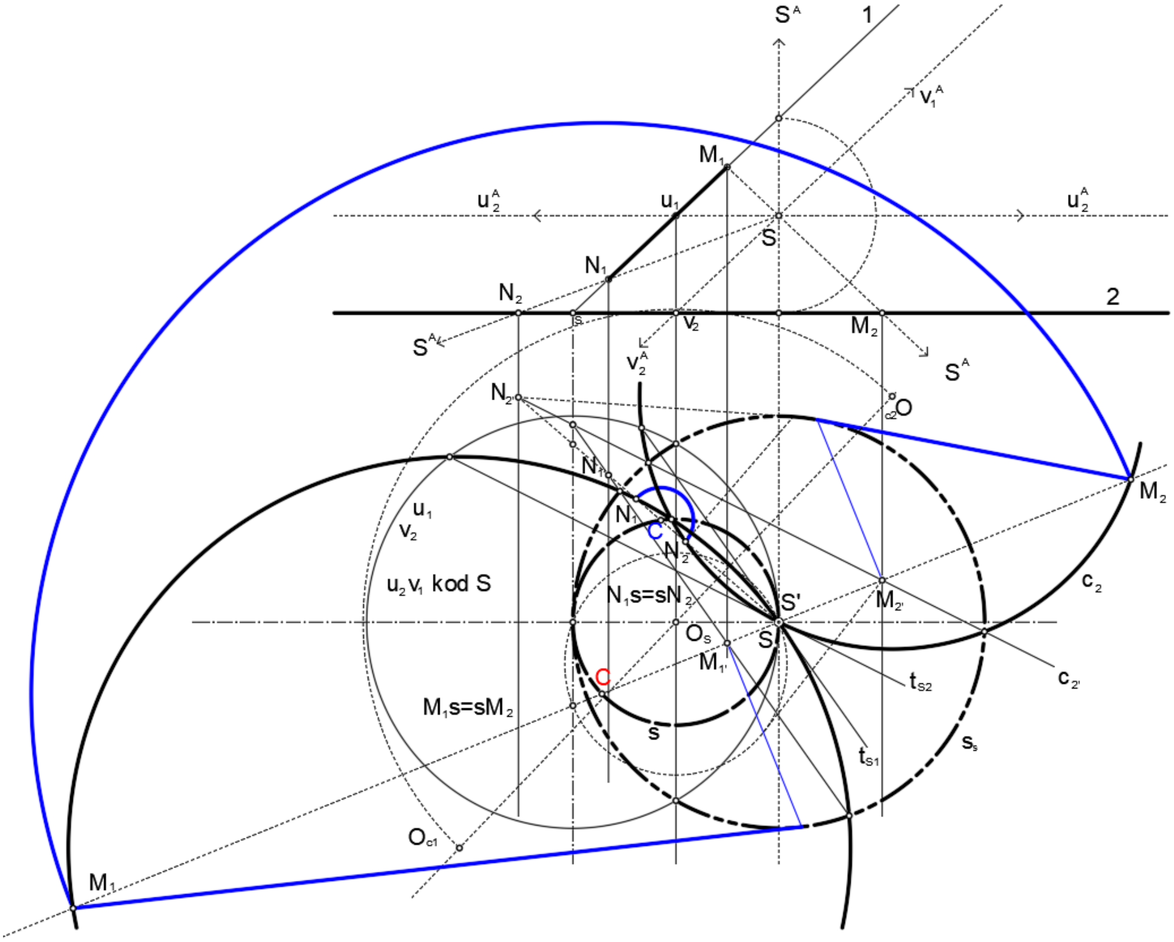

Fig. 5. Symmetry mapping of the points for center S and circular axis s (S, s)

By nonconformal symmetry, a pencil of hyperbolas is mapped into a pencil of curves of the eighth order (Fig. 6). In relativistic geometry, the hyperbola is of the fourth order (2+2A), while the obtained curve belongs to the eight order (6+2A) with a double point in S. The double point in the antipodal centre of symmetry is common to both curves, and therefore their corresponding asymptotes are parallel. In Fig. 7, a pencil of curves of the eight order is mapped into a pencil of curves of a higher order. There are no more infinite points in the pencil and everything unfolds before the eyes of the observer [1].

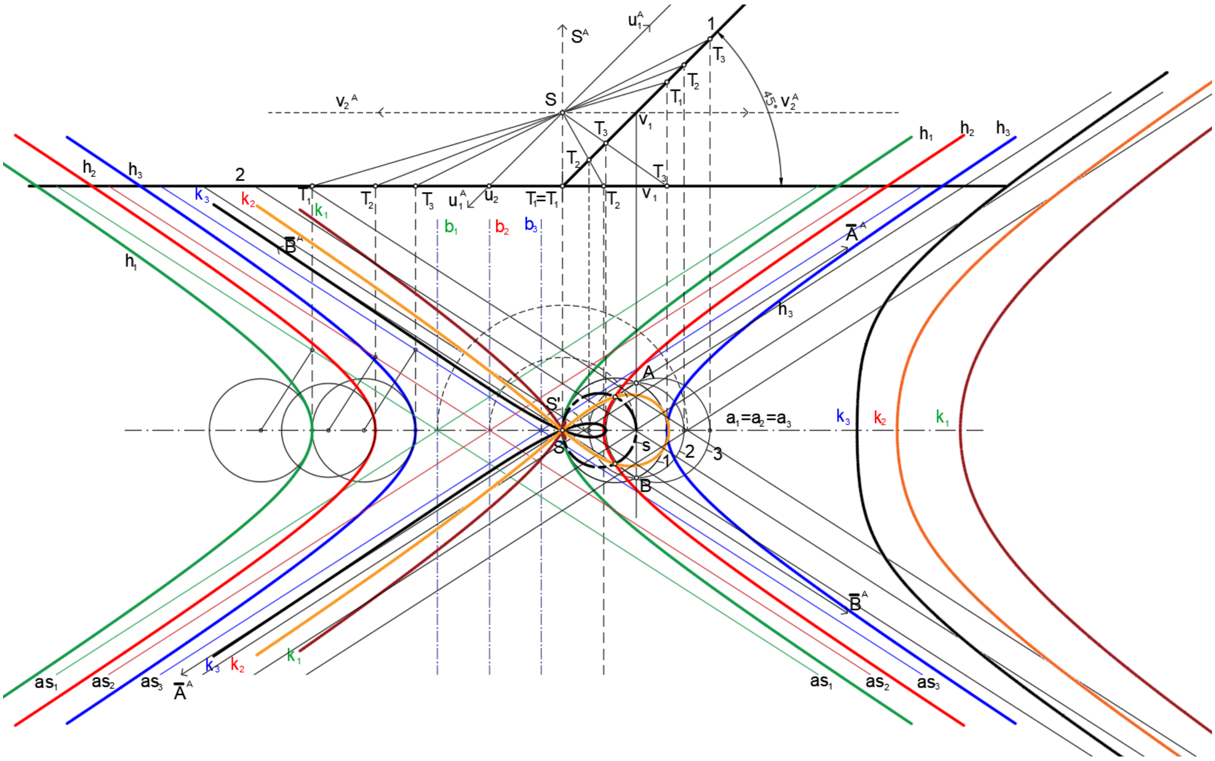

Fig. 6. Mapping of a pencil of hyperbolas into a pencil of curves of the eighth order (S,s) (relativistic order)

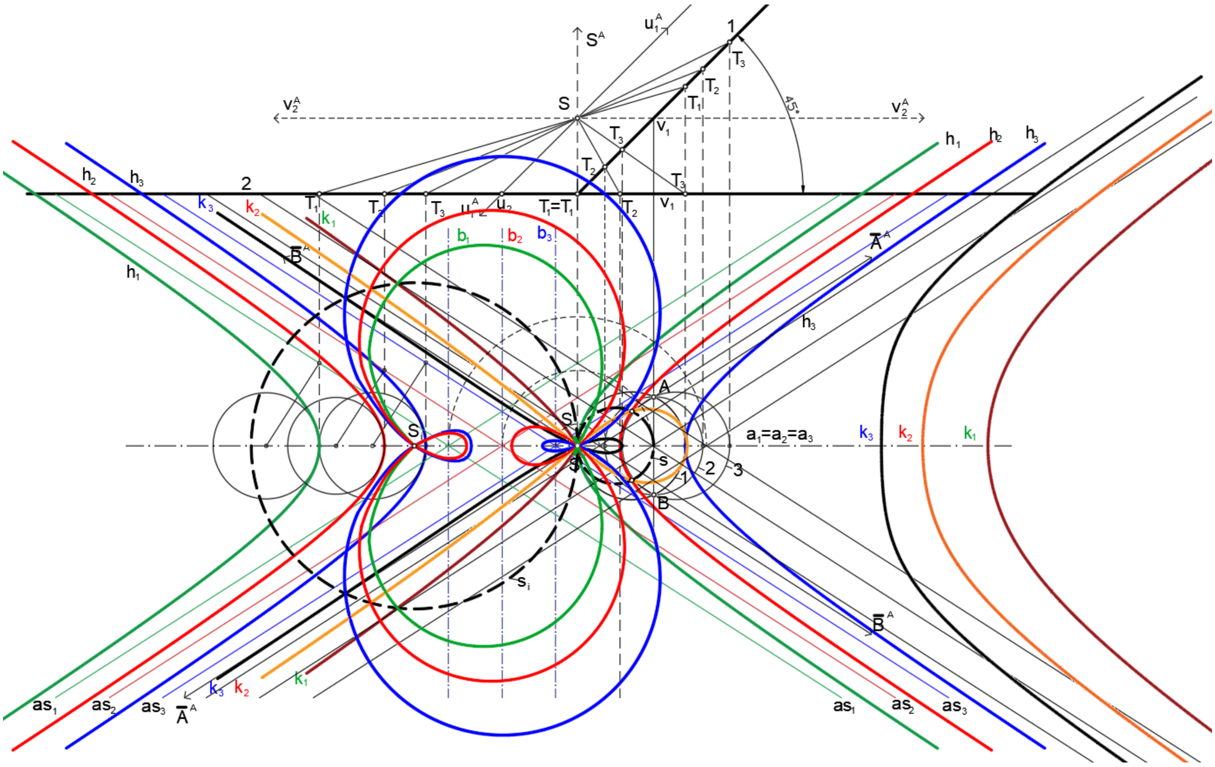

Fig.7. Inversion mapping of a pencil of curves of the eighth order into a pencil of curves of a higher order (Si, si)

2.2 Elliptic-parabolic (EP) pencil of conics

In the following example (Fig. 8), an elliptic pencil of circles, which is placed towards vanishing line v_1, is mapped by homology (perspective collineation) into a pencil of conics that contains an ellipse (circle 1 does not intersect the vanishing line), a parabola (circle 2 touches the vanishing line), a hyperbola (circle 3 intersects the vanishing line). The angle between planes 1 and 2 is 45˚. By perspective collineation, a pencil of conics is obtained from an elliptical pencil of circles. This pencil of conics is elliptic-parabolic (EP) because it has two real and separated points in (A and B) and two infinite, i.e. two overlapping points at the antipodal point.

The disintegrated conic of the pencil is represented by two straight lines that intersect in S ($S_{\overline{A}}$ and $S_{\overline{B}}$). They map into themselves like mapping rays.

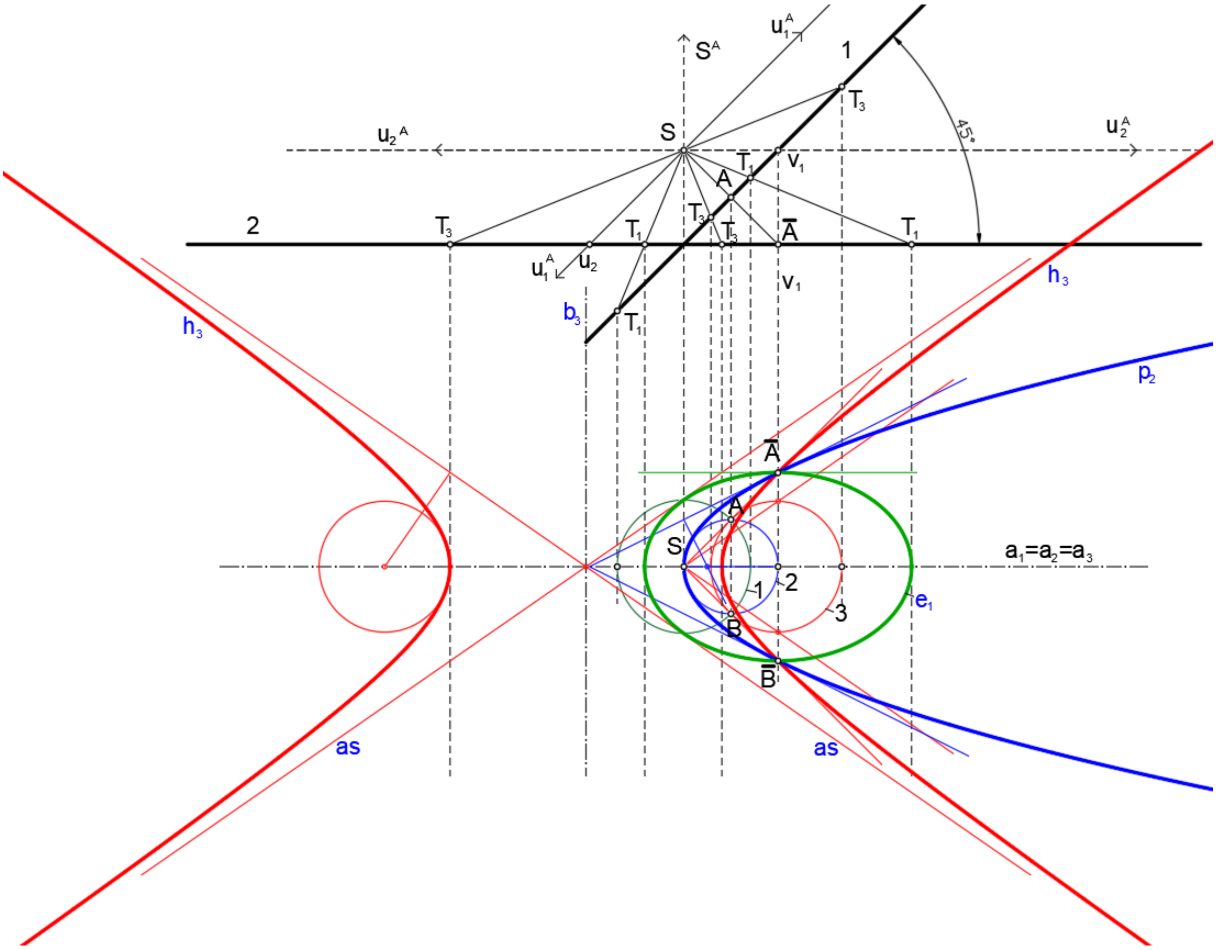

Fig. 8. Elliptic-parabolic (EP) pencil of conics

By nonconformal symmetry (Fig. 9) for the centre S and the circular axis s, we produced a pencil of mapped curves of the eighth order (relativistic order). This pencil contains elliptic, hyperbolic and parabolic curves. The points $\overline{A}$ and $\overline{B}$ and are mapped into point S. All the curves have two common overlapping points in S, and two points are at the antipode. The hyperbola maps into an eighth-order curve with a pair of asymptotes parallel to the asymptotes of the starting hyperbola. Only the centre is mapped by supersymmetry, which is shown by a semicircle with a dashed line.

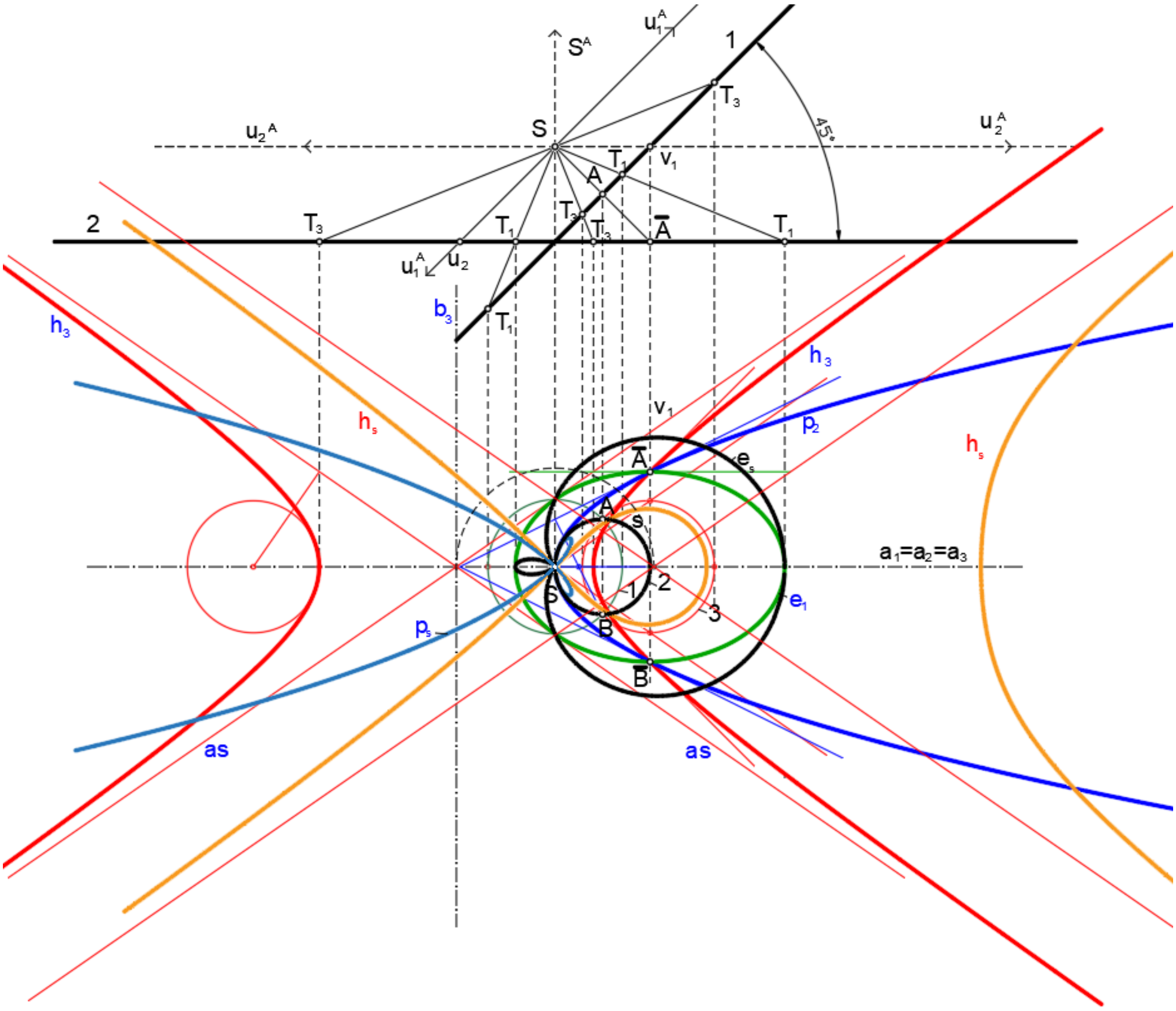

Fig. 9. Mapping of EP pencil of conics into EP pencil of curves using nonconformal symmetry for the center S and the circular axis s

3 CONCLUSIONS

This The three most important symmetries according to Dovniković L.:

- Relativistic inversion (conformal) was applied in the paper to obtain a pencil of higher-order curves.

- Relativistic harmonic homology (nonconformal) was applied to map pencils of circles.

- Supersymmetry i.e. inverted harmonic homology (nonconformal), which is the subject of this paper. It was used to map the pencils of the conics.

Recognizing the equivalence of inversion with classical axial symmetry (nonconformal symmetry) pointed to unlimited possibilities for mapping curves and obtaining new shapes that will be useful in the theory of geometry. Research in the area of symmetry will continue, which provides an inexhaustible space for further discoveries about planar and spatial curves. Curves can be mapped multiple times by symmetry, and all resulting curves will have the same properties as the original curve and will provide a wealth of desired shapes.

The nonconformal symmetry in relation to the circle, as an inverse image of the relativistic homology, enables us to perform the process of tying and untying of singular points, which in homology (by projecting the vanishing line into the antipodal circular point) ended in the "infinitely" distant antipode. It is now performed before the eyes of the observer (Fig. 7). Unlike inversion as conformal symmetry, which changes only the shape of the curve and nothing more, non-conformal symmetry can (but may not) change almost everything related to the number and type of singular points [8].

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