## GEOMETRY, GRAPHICS AND DESIGN IN THE DIGITAL AGE

The 9th International Scientific Conference on Geometry and Graphics

## MONGEOMETRIJA 2023

# Text formating 

Number of copies:

ISBN
978-86-6022-575-9

## Disclaimer

Authors are responsible for all pictures, contents and copyright-related issues in their own paper(s).

## ORGANIZING INSTITUTIONS

Serbian Society for Geometry and Graphics


Faculty of Technical Sciences


International Society for Geometry and Graphics


Digital Design Center


## SUPPORTED BY

Provincial Secretariat for Higher Education and Scientific Research


Ministry of Science, Technological Development and Innovations


Faculty of Technical Sciences


## ORGANIZING COMMITTEE

| Ivana Bajšanski | Faculty of Technical Sciences, University of Novi Sad | Serbia |
| :--- | :--- | :--- |
| Marko Jovanović | Faculty of Technical Sciences, University of Novi Sad | Serbia |
| Vesna Stojaković | Faculty of Technical Sciences, University of Novi Sad | Serbia |
| Bojan Tepavčević | Faculty of Technical Sciences, University of Novi Sad | Serbia |
| Marko Vučić | Faculty of Technical Sciences, University of Novi Sad | Serbia |
| Slobodan Mišić | Faculty of Applied Arts, University of Arts in Belgrade | Serbia |

## ASSISTANT TEAM

Jelena Kićanović
Miloš Obradović
Jelena Pepić

Faculty of Technical Sciences, University of Novi Sad
Faculty of Technical Sciences, University of Novi Sad
Faculty of Technical Sciences, University of Novi Sad

Serbia
Serbia
Serbia

## SCIENTIFIC COMMITTEE

| Vera Viana | Faculty of Architecture, University of Porto | Portugal |
| :--- | :--- | ---: |
| Marija Jevrić | Faculty of Civil Engineering University of Montenegro | Montenegro |
| Alina DUTA | University of Craiova | Romania |
| Ivana Vasiljević | Faculty of Technical Sciences, University of Novi Sad | Serbia |
| Tashko Rizov | University Ss. Cyril and Methodius Skopje, Faculty of <br> Mechanical Engineering Skopje | North Macedonia |
| Djordje Djordjevic | University of Belgrade-Faculty of Architecture | Serbia |
| Hellmuth Stachel | Vienna University of Technology | Austria |
| Ljiljana Radovic | Faculty of Mechanical Engineering, University of Nis | Serbia |
| Ema Jurkin | University of Zagreb, Faculty of Mining, Geology and | Croatia |
| Senonja Krasić | Faculty of Civil Engineering and Architecture in Nis | Serbia |
| Ksenija Hiel | Faculy of Agriculture Novi Sad, University of Novi Sad | Serbia |
| Dejana Nedučin | Faculty of Technical Sciences, University of Novi Sad | Serbia |
| Albert Wiltsche | Graz University of Technology | Austria |
| Ljubica Velimirovic | Faculty of Sciences and Mathematics, University of Nis | Serbia |


| Emil Molnar | Institute of Mathematics, Budapest University of Technology and Economics | Hungary |
| :---: | :---: | :---: |
| Vesna Stojakovic | Faculty of Tecnical Sciences, University of Novi Sad | Serbia |
| Marko Lazić | University of Novi Sad, Faculty of technical sciences, Department of Architecture | Serbia |
| Bojan Tepavčević | University of Novi Sad, Faculty of Technical Sciences | Serbia |
| Maja Petrović | University of Belgrade - The Faculty of Transport and Traffic Engineering | Serbia |
| Radovan Štulić | Faculty of Technical Sciences, University of Novi Sad | Serbia |
| Sofija Sidorenko | Ss. Cyril and Methodius University in Skopje, Faculty of Mechanical Engineering | North Macedonia |
| Ratko Obradovic | Faculty of Tecnical Sciences, University of Novi Sad | Serbia |
| Magdalena Dragovic | Faculty of Civil Engineering, University of Belgrade | Serbia |
| Milena Krklješ | University of Novi Sad, Faculty of Technical Sciences | Serbia |
| Slobodan Mišić | University of Arts in Belgrade, Faculty of Applied Arts | Serbia |
| Carmen Marza | Technical University of Cluj-Napoca | Romania |
| Zorana Jeli | Facultyof Mechanical Engineering, University of Belgrade | Serbia |
| Marija Obradović | Faculty of Civil Engineering, University of Belgrade | Serbia |
| Branislav <br> Popkonstantinović | Faculty of Mechanical Engineering, University of Belgrade | Serbia |
| Gordana Đukanović | University of Belgrade, Faculty of Forestry | Serbia |
| Milena Stavric | Graz University of Technology | Austria |
| Branko Malesevic | University of Belgrade, School of Electrical Engineering | Serbia |
| Honorable member |  |  |
| Lazar Dovniković | Faculty of Tecnical Sciences, University of Novi Sad | Serbia |

## ACKNOWLEDGEMENTS

Firstly, we want to express our gratitude to all of the authors whose papers are included in the present proceedings, as well as the chairs of the sessions, workshop instructors, the workshop organizers and the keynote speakers, whose contributions can be regarded critical to the success of this conference. We are also grateful to the international Scientific Committee who evaluated abstracts and final papers.

We are thankful to our local organizational team Vesna Stojaković, Bojan Tepavčević, Slobodan Mišić, Marko Vučić, Jelena Pepić, Jelena Kićanović and Miloš Obradović for their dedicated collaboration. We also wish to thank Rade Lučić from the IT center and other staff members at Faculty of Technical Sciences. We would like to thank University of Novi Sad, which allowed the conference events at the main Rectorate building.

Novi Sad, May 2023
Ivana Bajšanski and Marko Jovanović

# GEOMETRY, GRAPHICS AND DESIGN IN THE DIGITAL AGE 

Proceedings
The 9 ${ }^{\text {th }}$ International Scientific Conference on Geometry and Graphics
MoNGeometrija 2023

## Table of Contents

Jovana Tomić, Sonja Krasić, Nastasija Kocić
Optimization and rationalization in the design of floating settlements ..... 1
Tanja Mitrović, Milica Vračarić, Vesna Stojaković
Visibility analysis of urban spaces: Temporary structure layouts related to the quality of urban seating ..... 7
Marko Vučić, Bojan Tepavčević
Design and fabrication of depolluting façade system elements ..... 15
Ina Pašić, Marko Jovanović
Design and simulation of concrete panels cast in stencil constrained fabric formwork ..... 25
Marko Lazić, Ana Perišić
Algorithm for geometry optimization of complex building floorplan footprints into a grid of quads ..... 33
Milica Pavlović, Marko Jovanović
A case study of acoustic diffusers impact on echo reduction in auditoriums ..... 43
Nastasija Kocić, Branislava Stoiljković, Sonja Krasić, Jovana Tomić
Rationalization of ellipsoidal shell for prefabrication ..... 55
Tashko Rizov, Aleksandar Jankovic, Risto Tashevski, Elena Angeleska
Visualisation of autonomouos vehicle interior in virtual reality ..... 61
Sanja Dubljević, Jelena Kićanović, Aleksandar Anđelković
Integration of building information modeling (BIM) and virtual reality technology (VR) for daylight analysis visualization ..... 69
Dragoș - Laurențiu Popa, Cosmin Berceanu, Gabriel Catalin Marinescu, Violeta Contoloru, Duță Alina, Daniela Doina Vintilă, Daniel Cosmin Calin, Gabriel Buciu
Virtual simulations of some human anatomical systems ..... 79
Miloš Obradović
Various concepts of user movement within the immersive virtual space in architecture ..... 93
Jelena Milošević, Ljiljana Đukanović, Milijana Živković, Maša Žujović, Marko Gavrilović
Automated compositions: Artificial intelligence aided conceptual design explorations in architecture ..... 103
Jelena Pepić
Application of digital tools in the fabrication of tensile structures ..... 117
Dirk Huylebrouck
A new family of solids: The infinite kepler-poinsot polyhedra ..... 125
Gordana Đukanović, Đorđe Đorđević, Mirjana Devetaković, Đorđe Mitrović
Transformation of pencils of circles into pencils of conics and these into pencils of higher- order curves ..... 133
Branislav Popkonstantinović, Miša Stojicević, Ratko Obradović, Ivana Cvetković
The gravity escapement - It all boils down to geometry ..... 143
Đorđe Mitrović, Đorđe Đorđević, Mirjana Devetaković, Gordana Đukanović
Encoding/decoding capitals of classical architectural orders by using fractal geometry: Establishing methodology ..... 153
Marija Obradović, Slobodan Mišić
Learning while playing - Througie platform for creating models of spatial structures ..... 169
Aleksandra Stakić, Zorana Jeli, Nedeljko Dučić, Boris Kosić
An overview of the classification of four-bar mechanisms in view of the motion simulation in the matlab software package ..... 181
Maja Petrović, Dragan Lazarević, Aleksandar Trifunović, Branko Malešević
Proposal of new constant slope surfaces for the purposes of designing traffic infrastructure elements ..... 195
Marija Obradović, Anastasija Martinenko
A method for adjusting the shape of semi-oval arches using Hügelschäffer's construction ..... 205
Hellmuth Stachel
The design of skew gears from the geometric point of view ..... 217
Emil Molnár, Jenő Szirmai
On SL2R crystallography ..... 229
Aleksandra Bobić, Marko Jovanović, Ivana Vasiljević
Environmental geometry generation in video games using photogrammetry and digital sculpting ..... 247
Ivana Miškeljin, Igor Maraš, Marko Todorov
Experimental approaches to architectural visualization: Learning from visual arts ..... 257
Milan Miščević, Ivana Vasiljević, Ratko Obradović
Types of level design structures in video game development ..... 271
Vladan Nikolić, Olivera Nikolić, Jasmina Tamburić, Sanja Spasić Đorđević, Jovana Vukanić Application of composite (hybrid) graphics in architectural representation ..... 279
Isidora Mitrović
Reconstruction of the building using photogrammetry method - Case study of the chapel of st. George on mountain Rtanj ..... 289
Isidora Đurić, Miloš Obradović, Vesna Stojaković
Free software for image-based modeling education: Comparative analysis - advantages and disadvantages ..... 297
Mirjana Devetaković, Đorđe Đorđević, Nikola Popović, Đorđe Mitrović, Gordana Đukanović, Slobodan Mišić
On the other side of mirror - a workshop on incorporating geometry of mirroring in architectural practice and applied arts ..... 309
Ludmila Sass, Alina Duta, Anca Barbu, Alina Elena Romanescu
Contributions to the study of da Vinci 's lunules ..... 323
Sonja Krasić, Jovana Tomić, Nastasija Kocić, Zlata Tošić
Online teaching on a cademic course descriptive geometry at the Faculty of civil engineering and architecture in Niš ..... 335
Naomi Ando
Application of bim to architectural design education ..... 345
Domen Kušar, Mateja Volgemut
The correlation between students' spatial perception and academic success in the descriptive geometry course ..... 357
Miša Stojićević, Branislav Popkonstantinović, Zorana Jeli, Ivana Cvetković, Boris Kosić Analysis of a ptc systems with moving focal point ..... 371
Dragan Lazarević, Momčilo Dobrodolac, Maja Petrović, Aleksandar Trifunović
Application of geometric modeling to improve the efficiency of the delivery phase in the e- commerce ..... 381
Biljana Jović, Aleksandar Čučaković, Marija Marković , Katarina Bašić
Sustainable solar lamp biodesign inspired by the crocus vernus I. Flower ..... 389
Cosmin Berceanu, Alina Duță, Dragoș Popa, Anca Didu
Considerations regarding the shape definition of an anthropomorphic robotic hand-arm system ..... 399
Boris Kosić, Zorana Jeli, Aleksandra Stakić, Marko Rusov, Dragoljub Bekrić, Zaga Trišović
Analisys of a four-bar linkage complaint mechanism for receiving approximately straight-linemotion407
Milan Stojanović, Pavle Ljubojević, Tatjana Lazović
Simulation of involute gear tooth profile shaping ..... 417
Gabriel Cătălin Marinescu, Ludmila Sass, Anca Didu, Oana Victoria Oțăt
Modeling and analysis of some braking system parts ..... 429

# TRANSFORMATION OF PENCILS OF CIRCLES INTO PENCILS OF CONICS AND THESE INTO PENCILS OF HIGHER-ORDER CURVES 

Gordana Djukanovic ${ }^{1 *}$, Djordje Djordjevic ${ }^{2}$, Mirjana Devetakovic ${ }^{3}$, Djordje Mitrovic ${ }^{4}$<br>${ }^{1}$ Associate Professor PhD, Faculty of Forestry, University of Belgrade (SERBIA), gordana.djukanovic@sfb.bg.ac.rs<br>${ }^{2}$ Associate Professor PhD, Faculty of Architecture, University of Belgrade (SERBIA), djordje@arh.bg.ac.rs<br>${ }^{3}$ Assistant Professor PhD, Faculty of Architecture, University of Belgrade (SERBIA), mirjana.devetakovic@gmail.com<br>${ }^{4}$ PhD Student, Faculty of Architecture, University of Belgrade (SERBIA), djordje.mitrovic@arh.bg.ac.rs


#### Abstract

In this paper, an elliptical pencil of circles is mapped by homology (perspective collineation) into a parabolic-parabolic pencil of hyperbolas because it has two by two common points $1=2$ and $3=4$ in infinity, i.e., four overlapping points in the antipode. The mapping also includes the mapping of degenerated conics decomposed into corresponding pairs of straight lines. The elliptical pencil of circles (the ellipses on plane 1) is mapped into a hyperbolic pencil (on plane 2) by perspective collineation for pole $\mathrm{S}_{1}$. The vanishing line $\mathrm{v}_{1}$ intersects all the circles of the pencil so that by homology all the circles are mapped into hyperbolas. To apply supersymmetry to the obtained pencils, the angle between plane 1 and plane 2 must be $45^{\circ}$. The pencil of hyperbolas is mapped by supersymmetry into a pencil of higher-order curves. The obtained pencils are further mapped by inversion and then again by supersymmetry to obtain higher-order curves rich in different shapes.

In the second example, the elliptical pencil of circles is placed to the vanishing line $\mathrm{v}_{1}$ so that by homology, it is mapped into a pencil of conics containing an ellipse (circle 1 does not intersect the vanishing line), a parabola (circle 2 touches the vanishing line), a hyperbola (circle 3 intersects the vanishing line). The pencil of conics is elliptical-parabolic because it has two real and separate points and two infinite, i.e., two overlapping points in the antipode. This pencil of conic is also mapped by supersymmetry into a pencil of higher-order curves that intersect at the same number and type of base points as the starting pencil. Higher order curves obtained by mapping are rational line curves.


Keywords: pencils of circles, pencils of conics, inversion, supersymmetry, pencils of curves of the higher orders

## 1 INTRODUCTION

Professor Lazar Dovnikovic has made a significant contribution to the study of curves. Modern geometry has introduced the term observer (hence the term relativistic geometry) [3]. The term "plane" is replaced by "sphere" and the term "straight line" by "circle passing through the observer's antipodal point". Perception and interpretation of geometric elements depend solely on the observer's position. Through the observer's standing point on the "plane", a pencil of "straight" geodesics cuts through. Each of these largest circles on the sphere defines a pencil of straight lines that are parallel to them but are not geodesics. By moving away from the geodesics on both sides, the diameter of the circles decreases. (Fig. 1). For each direction at the antipode, there is one infinitesimal circle. Thus, we come to an unusual conclusion: Unlike the projective plane which has only one infinitely distant straight line for all observers, the relativistic "plane" has infinitely many infinitesimal antipodal "straight lines" for each (of $\infty^{2}$ ) observer individually. However strange it may be, this fact paves the way for creating a simple mechanism for the tying and untying singular points of curves.

A simple example in Fig. 1 shows the essence of relativistic geometry. It explains the concept of parallel lines. Regarding two parallel lines, projective geometry introduces the concept of an infinitely distant point where these two parallel lines intersect. Since parallel lines are equidistant, they can never meet, not even at infinity. Relativistic geometry explains the concept of parallel lines by the fact that an observer standing on a large sphere, on the geodesic marked with d (Fig. 1,d coloured black), perceives all the circles that pass through his antipodal point as parallel or intersecting lines. In the given example, the "straight lines" are parallel to the "d" geodesic of the sphere. By stereographic projection of circles from the antipodal point $S_{A}$ (red and green circles shown in Fig. 1) onto plane $T$, a pencil of parallel lines (blue lines) is obtained. Therefore, the observer perceives the red and green circles on his large sphere as blue parallel lines.


Fig. 1. Parallel lines in relativistic geometry

In relativistic geometry, curves are classified into harmonic groups. Fig. 2 shows the regrouping of curves according to the classical and relativistic order. Figure 1 was taken from "Quantum-relativistic geometry as a new scientific paradigm" [8] .
The relativistic order of the curve equals the sum of its classical order and the number of times the curve passes through the antipode.

The "straight line" and circle are second-order curves because a "straight line" is a circle passing through the observer's antipodal point. Classical second-order curves have a double point at the antipode, which makes them fourth-order curves in relativistic geometry. The curves of the fourth order of the elliptical type (Fig. 2) have a double isolated point at the antipode. Hyperbolic curves have a double self-intersecting point at the antipode, and parabolic curves have a cusp where the two branches of the curve touch. When the laws of symmetry are applied to curves in a certain group, the properties of higher-order curves will depend on the leading curve in that group. The classification of curves into harmonic groups is of great value in the theory of geometry [4].

## 2 TRANSFORMATION OF AN ELLIPTICAL PENCIL OF CIRCLES

The following segment shows the mapping of elliptic pencils of circles into corresponding pencils of conics by homology (perspective collineation) which are then mapped by symmetry (for the centre $S$ and the circular axis s) into pencils of higher-order curves. An inversion is then applied to the obtained curves (for the centre $\mathrm{S}_{\mathrm{i}}$ and the circular axis $\mathrm{s}_{\mathrm{i}}$ ) [2] .

### 2.1 Parabolic-Parabolic (PP) pencil of conics

The elliptical pencil of circles (i.e. ellipses on plane 1 seen as circles in the first projection) which is projected into (2) elliptical pencil of hyperbolas with two pencils of parallel asymptotes is shown in Fig. 3.


Fig. 2. Classical and relativistic order of curves

Three circles, marked 1, 2 and 3 , are mapped into hyperbolas $h_{1}, h_{2}$ and $h_{3}$. Vanishing line $v_{1}$ intersects all the circles of the pencil, so all circles are mapped into hyperbolas using perspective collineation. The pencil of hyperbolas is parabolic-parabolic (homothetic hyperbolas) because it has two pairs of common points $1=2$ and $3=4$ at infinity, i.e. four points overlapping at the antipodal point. The disintegrated conic of the pencil is represented by two straight lines that intersect in S (SA and SB). Like mapping rays that pass through S, they are mapped into themselves. The mapped points $\overline{\mathrm{A}}$ and $\overline{\mathrm{B}}$ are at the observer's antipode. The asymptotes of all hyperbolas are parallel to these rays. A

Lisp routine was written to draw the hyperbola [7]. For that purpose, the UCS (user coordinate system) is inserted into the centre of the hyperbola. The parameters a and $b$ of the hyperbola are loaded, with the condition that the minimum value along the axis $x$ must be $a$, and the maximum value along the axis $x$ is set at 150 mm (or more). A step of 0.5 mm is set to obtain the required quality of curve smoothness. To follow the mapping of hyperbolas, they are coloured with three different colours.


Fig. 3. Parabolic-parabolic pencil of hyperbolas


Fig. 4. Mapping of hyperbola numbered 3 into a curve of the eighth order ( $(S, s$ )

A Lisp program was written for drawing the mapped curves using nonconformal symmetry. First, the hyperbola numbered three was mapped into the curve of the eighth order ( $\mathrm{S}, \mathrm{s}$ ) [6]. The program was run by placing the UCS at the centre of symmetry S, loading the radius of the absolute and simply clicking on the curve to be mapped. To write the program for symmetry mapping, we created mathematical equations for the coordinates of the mapped points using the UCS coordinate system in centre S [10]. The following equations for the coordinates of the mapped points $\bar{P}\left(\frac{-}{x},-\frac{y}{y}\right)$ were used provided that the mapped point is $\mathrm{P}(\mathrm{x}, \mathrm{y})[5]$ :

$$
\bar{x}=\frac{4 r x^{2}}{x^{2}+y^{2}}-x \quad \bar{y}=\frac{4 r x y}{x^{2}+y^{2}}-y
$$

Applying axial symmetry, the mapping of points is performed simply by shifting the distance between point $P$ and circular axis $s$ to the other side in the direction of the centre of symmetry $S$.
Fig. 5 shows simple axial symmetry obtained by mapping points named $M_{1}$ and $N_{1}$. The points are in plane 1 and are mapped by perspective collineation through the centre $S$ into plane 2 (points $\mathrm{M}_{2}$ and $N_{2}$ ). Points $M_{1}$ and $M_{2}$ are then at the base point (for centre $S$ and inversion circle $s_{s}$ ) inverted into points $M_{1}$ and $M_{2}$. Points $N_{1}{ }^{\prime}$ and $N_{2}{ }^{\prime}$ are also inverted into points $N_{1}$ and $N_{2}$. The points on the absolute which are used to shift the distance of the points from the absolute to the other side of the centre $S$ are marked with the red letter $C$ for $M_{1}$ and $M_{2}$ and with the blue letter $C$ for $N_{1}$ and $N_{2}$. The large blue semicircle shows the obtaining of point $M_{2}$ by mapping symmetry mapping from point $M_{1}$. The smaller blue semicircle shows obtaining of point $N_{2}$ by mapping point $N_{1}$. The radius of the absolute - s in axial symmetry is equal to half the radius of the circle of inversion $-\mathrm{s}_{\mathrm{s}}$, and the centres of $S$ for inversion and symmetry coincide [9].


Fig. 5. Symmetry mapping of the points for center $S$ and circular axis $s(S, s)$

By nonconformal symmetry, a pencil of hyperbolas is mapped into a pencil of curves of the eighth order (Fig. 6). In relativistic geometry, the hyperbola is of the fourth order ( $2+2 \mathrm{~A}$ ), while the obtained curve belongs to the eight order $(6+2 A)$ with a double point in $S$. The double point in the antipodal centre of symmetry is common to both curves, and therefore their corresponding asymptotes are parallel. In Fig. 7, a pencil of curves of the eight order is mapped into a pencil of curves of a higher order. There are no more infinite points in the pencil and everything unfolds before the eyes of the observer [1].


Fig. 6. Mapping of a pencil of hyperbolas into a pencil of curves of the eighth order ( $\mathcal{S}, \mathrm{s}$ ) (relativistic order)


Fig.7. Inversion mapping of a pencil of curves of the eighth order into a pencil of curves of a higher order (Si, si )

### 2.2 Elliptic-parabolic (EP) pencil of conics

In the following example (Fig. 8), an elliptic pencil of circles, which is placed towards vanishing line $\mathrm{v}_{1}$, is mapped by homology (perspective collineation) into a pencil of conics that contains an ellipse (circle 1 does not intersect the vanishing line), a parabola (circle 2 touches the vanishing line), a hyperbola (circle 3 intersects the vanishing line). The angle between planes 1 and 2 is $45^{\circ}$. By perspective collineation, a pencil of conics is obtained from an elliptical pencil of circles. This pencil of conics is elliptic-parabolic (EP) because it has two real and separated points in (A and B) and two infinite, i.e. two overlapping points at the antipodal point.
The disintegrated conic of the pencil is represented by two straight lines that intersect in $S^{\left(S_{\bar{A}}\right.}$ and $S_{\overline{\mathrm{B}}}$ ). They map into themselves like mapping rays.


Fig. 8. Elliptic-parabolic (EP) pencil of conics

By nonconformal symmetry (Fig. 9) for the centre $S$ and the circular axis $s$, we produced a pencil of mapped curves of the eighth order (relativistic order). This pencil contains elliptic, hyperbolic and parabolic curves. The points $\bar{A}$ and $\bar{B}$ and are mapped into point $S$. All the curves have two common overlapping points in S, and two points are at the antipode. The hyperbola maps into an eighth-order curve with a pair of asymptotes parallel to the asymptotes of the starting hyperbola. Only the centre is mapped by supersymmetry, which is shown by a semicircle with a dashed line.


Fig. 9. Mapping of EP pencil of conics into EP pencil of curves using nonconformal symmetry for the center $S$ and the circular axis s

## 3 CONCLUSIONS

This The three most important symmetries according to Dovniković L.:

- Relativistic inversion (conformal) was applied in the paper to obtain a pencil of higher-order curves.
- Relativistic harmonic homology (nonconformal) was applied to map pencils of circles.
- Supersymmetry i.e. inverted harmonic homology (nonconformal), which is the subject of this paper. It was used to map the pencils of the conics.
Recognizing the equivalence of inversion with classical axial symmetry (nonconformal symmetry) pointed to unlimited possibilities for mapping curves and obtaining new shapes that will be useful in the theory of geometry. Research in the area of symmetry will continue, which provides an inexhaustible space for further discoveries about planar and spatial curves. Curves can be mapped multiple times by symmetry, and all resulting curves will have the same properties as the original curve and will provide a wealth of desired shapes.
The nonconformal symmetry in relation to the circle, as an inverse image of the relativistic homology, enables us to perform the process of tying and untying of singular points, which in homology (by projecting the vanishing line into the antipodal circular point) ended in the "infinitely" distant antipode. It is now performed before the eyes of the observer (Fig. 7). Unlike inversion as conformal symmetry, which changes only the shape of the curve and nothing more, non-conformal symmetry can (but may not) change almost everything related to the number and type of singular points [8].


## 4 ACKNOWLEDGMENT

This research has been completed within the following two research projects: 451-03-47/202301/200169 i 451-03-47/2023-01/200090, funded by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia.

## REFERENCES

Referencing to articles that have appeared in periodicals:
[1] Dovniković, L (2004). Relativistic Homology as a Way of Tying or Untying Singular Points, Journal for Geometry and Graphics, Vienna, Volume 8, No.2, 2004, pp 151-162
[2] Đukanović, G. et al (2012). The Pencil of the 4thand 3rd Order Suraces Obtained as a Harmonic Equivalent of the Pencil of Quadrics throught a 4th Order Space curve of the 1st Category, Facta Universitatis, Series: Architecture and Civil Engineering, VOL. 10 N0 2.2012, pp. 193-207.

Referencing to books:
[3] Dovniković L. (1999). The Harmony of the Spheres, The Relativistic Geometry of Harmonic Equivalents, Matica srpska, Novi Sad.
[4] Dovniković, L. (1977). Descriptive Geometrical Treatment And Classification Of Plane Curves Of The Third Order - Doctoral disertation, Matica Srpska, Novi Sad.
[5] Đukanović, G. (2012). The Pencils of Curves of the Third and Fourth Order obtained by Mapping the Pencils of Conics, Doctoral dissertation, Faculty of Architecture, Belgrade.
[6] Nicholas, M. P. et al (2009). Shape Interrogation for Computer Aided Design and Manufacturing, Springer Science \& Business Media

Referencing to Papers that have appeared in a long-running series of various Archives, even if they appeared on CD-ROM firstly or as to a loose Papers at the Workshops, Symposiums or Congresses
[7] Benton, B. (2011). Advanced Auto-CAD 2011 Training DVD-Tutorial, Infinite Skills.
Referencing to edited Conference Proceedings:
[8] Dovniković, L. (2010). Quantum-Relativistic Geometry as a new Scientific Paradigm, Proceedings of 2nd international Scientific Conference moNGeometrija 2010, Beograd
[9] Đukanović, G. et al (2014). Mapping of the Pencils of Conics Using Supersymmetry - Inversion of Harmonic Homology, 4th International Scientific Conference on Geometry and Graphics moNGeometrija 2014, proceedings volume 2. str.180-193, June 20th - 22nd, Vlasina, Serbia
[10] Đukanović, G. et al (2020). Obtaining pencils of curves of higher order by applying a supersymmetry to pencils of conics, Proceedings of 7th International Scientific Conference moNGeometrija 2020., Belgrade, pp. 261-268.

CIP - Каталогизација у публикацији
Библиотеке Матице српске, Нови Сад
514.18(082)
004.92(082)
7.05:004.92(082)

INTERNATIONAL Scientific Conference on Geometry, Graphics and Design in the Digital Age (9 ; 2023 ; Novi Sad)
Proceedings / The 9th International Scientific Conference on Geometry, Graphics and Design in the Digital Age, MoNGeomatrija 2023, June 7-10, 2023, Novi Sad, Serbia ; editors Ivana Bajšanski, Marko Jovanović. - Novi Sad : Faculty of Technical Sciences ; Belgrade : Serbian Society for Geometry and Graphics (SUGIG), 2023 (Novi Sad : Grid). - 457 str. : ilustr. ; 30 cm

Tiraž 10. - Bibliografija uz svaki rad.

ISBN 978-86-6022-575-9
а) Нацртна геометрија -- Зборници б) Рачунарска графика -- Зборници в) Дигитални дизајн -Зборници

COBISS.SR-ID 116382985

