The 21st SEFI Special Interest Group in Mathematics – SIG in Mathematics

June 11th to June 14th 2023

Tampere University of Applied Sciences (TAMK)

Tampere, Finland

PROCEEDINGS

Tampere University of Applied Sciences

SEFI European Society for Engineering Education

STEM CENTER TAMK
Physics Chemistry Mathematics Technology Pedagogy

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INTRODUCTION

The Steering Committee of the SEFI Mathematics Special Interest Group is pleased to present the proceedings of the 21st SEFI MSIG Seminar on Mathematics in Engineering Education. This seminar was locally organized by the Tampere University of Applied Sciences, and has taken place from June 11st to 14th, 2023. Since its establishment in 1982, the SEFI Mathematics Working Group (MWG), now known as Mathematics Special Interest Group (MSIG), has consistently upheld its goals, which remain pertinent and applicable even after 41 years. These objectives include:

- Providing a platform for the exchange of perspectives and ideas among individuals interested in engineering Mathematics.
- Promoting a comprehensive understanding of the role of Mathematics in the Engineering curriculum and its significance in meeting industrial needs.
- Cultivating collaboration in the development of courses and supporting materials.
- Recognizing and advocating for the role of Mathematics in the ongoing education of engineers, in collaboration with the industry.

This seminar edition was based on mathematical competencies in didactical research with emphasis on practise roles of researchers and practitioners and their cooperation. How to assess competencies and the major aim of teaching Mathematics as a tool to future Engineers was also among the discussed topics.

Since 1984, the SEFI MSIG, former known as MWG, has organized 20 seminars on Mathematics in Engineering Education, dedicated to fulfilling its objectives and encouraging international participation. The 21st Seminar took place at the beautiful and welcoming city of Tampere, and marked the continuation of this successful series, bringing together passionate Mathematics educators. These seminars are designed to serve as a platform for the exchange of perspectives and ideas among participants interested in innovative methodologies in teaching Mathematics to Engineering students, i.e., participants that are interested in exploring and learning about new and creative approaches or techniques for teaching Mathematics specifically to Engineering students.

Its overarching goal is to advance a comprehensive understanding of the role of Mathematics in the Engineering curriculum, emphasizing its relevance to industrial needs and the ongoing education of Engineers within the European context. This seminar featured presentations and discussions on various crucial topics identified by the SEFI MSIG Steering Committee, as well as other pertinent issues in the Mathematical Education of Engineers. The central theme of the seminar revolves around the concept of mathematical competencies, encompassing the following themes:

- Mathematical competencies in practice and didactical research
- How to assess competencies?
- The goal of teaching
- Active learning strategies
- Mathematical Engineering and Informatics

Programme of the seminar included three plenary keynote lectures presented by excellent invited speakers, professors teaching mathematics at universities in different European countries.

Professor Markku Saarelainen from Tampere University, Finland spoke about “Flipped Classroom as a strategic approach in teaching – a brief history from theory to practice in Finnish university level “.

Professor Per Henrik Hogstad from the University of Agder, Norway, presented the talk on “Visualizations and simulations in mathematics and physics“, and finally

Professor Ion Mierlus-Mazilu from Bucharest Technical University of Civil Engineering, in Romania, discussed topic “The Use of Mathematical Software in Teaching Mathematics “.

Despite the limitations imposed by the global health crisis that we faced, your commitment to advancing knowledge, working with your students, and sharing insights in the field of engineering education has been perseverant. As a result of this perseverance, a positive response to the seminar's call for papers led to the acceptance of 23 high-quality submission directly aligned with the seminar themes, all addressing crucial topics in the mathematical education of engineering students.

It is important to acknowledge the unique circumstances that have shaped our experiences over the past couple of years. The pandemic has presented unprecedented challenges, disrupted traditional modes of education, and forced us to adapt to online platforms for learning and collaboration. It is a testament of dedication that all teachers have embraced this new normal, making the most of the virtual environment to engage in fruitful discussions and expand yours and your students' horizons. One of the key aspects we must explore is the significance of Mathematics in Engineering Education. Mathematics forms the foundation upon which Engineering principles and concepts are built. It provides the necessary tools for problem-solving, critical thinking, and analytical reasoning, all of which are essential for engineering students to excel in their chosen fields. Without a solid understanding of Mathematics, it would be challenging to comprehend and apply complex Engineering theories. However, it is equally important to strike a balance between the rigor of mathematical concepts and their practical applications. While Mathematics provides a theoretical framework, it is through practical classes and real-life problem-solving that students truly grasp the essence of Engineering. The application of Mathematical principles in practical scenarios helps students bridge the gap between theory and practice, fostering a deeper understanding of engineering concepts. Integrating ICT into Mathematics classes not only enhances students' engagement and understanding but also prepares them for a technology-driven world where mathematical skills are in high demand. It is important to strike a balance between mathematical rigor and the utilization of ICT tools, ensuring that students develop a deep understanding of mathematical concepts while leveraging technology to enhance their learning experience.

All accepted contributions are incorporated as full papers in the proceedings, which are freely accessible on the SEFI MSIG webpage. This initiative aims to offer a comprehensive overview of the seminar's topics and provide unrestricted access to the presented papers for all interested colleagues. The primary goals of the group are to uphold the ongoing process of collecting published materials and reports on all crucial topics identified in the mathematical education of engineers. This aims to construct a robust body of knowledge within this field. Lastly, the author expresses gratitude to all members of the SEFI Mathematics Special Interest Group Steering Committee, language editors, and local organizers for their efforts in conducting language checks and editing the proceedings. These contributions are intended to enhance the quality of the proceedings for the benefit of all potential readers.

In Coimbra, June 2023

Deolinda Dias Rasteiro
SEFI MSIG chair

Mathematical education of future architects – playing and learning with and in Iterated Function Systems

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Abstract

The principal methods of architectural design and construction have been inseparable from mathematics since time immemorial. Even nowadays, mathematical ability is an integral part of the professional competence of any architect, and therefore, mathematical education is naturally a very significant segment of architectural studies. At the University of Belgrade, the Faculty of Architecture, the leading higher education institution for architecture and urban studies in the Western Balkans region, undergraduate students primarily meet with mathematics on the core course Mathematics in architecture. As the name suggests, this course covers the basics of analytic and fractal geometry, focusing on application possibilities. Namely, after understanding and adopting the theoretical foundations using standard teaching methodology, it often happens that students hardly recognize the connection between the acquired mathematical knowledge and their future professional activities. Therefore, after each thematic unit during the course, some experimental work or functional research is carried out. This paper briefly presents an active learning method implemented on Iterated Function Systems (IFS), as one of the units. The accompanying experimental student task involves creating various analog models of IFS which could be interpreted architecturally.

Introduction

The use of mathematics in architecture has a long and rich history, dating back to ancient times when architects and builders utilized mathematical principles to create various structures that were both beautiful and functional. Even today, mathematical proficiency is essential for the professional competence of any architect, and therefore, mathematical education is a significant segment of architectural studies at the University of Belgrade's Faculty of Architecture, the leading higher education institution for architecture and urban studies in the Western Balkans region.

The first section of this paper provides a brief overview of the architectural studies at the Faculty, with a focus on classical mathematical courses as well as those that primarily rely on mathematical reasoning. While students acquire a solid theoretical foundation through standard teaching methodologies, they may not always grasp the connection between the mathematical knowledge they receive and their future professional activities. Therefore, it is not sufficient to only learn the theoretical foundations of mathematics. To be successful in their future careers, architecture students must also be able to apply their knowledge in practical ways. As such, our mathematical courses include experimental work or functional research after each thematic unit.

The subsequent sections of the paper concentrate on one such active learning method that has been implemented on Iterated Function Systems (IFS). This method has demonstrated significant effectiveness in helping students comprehend the relationship between their mathematical knowledge and their professional careers. The experimental work conducted through this method has resulted in the creation of numerous analog models of fractals that can be interpreted in an architectural context, some of which are presented in the paper.

Lastly, the paper concludes with insights on the described learning method and the approach to teaching mathematics for future architects, in general.

Mathematical education of future Serbian architects

The University of Belgrade's Faculty of Architecture is the premier institution for architectural and urban planning education in Serbia, with a history of over 170 years. The results achieved in the domains of education, professional and artistic creation and scientific research have ensured the Faculty's high-ranking position in the region. The Faculty provides a comprehensive education for future architects, offering a diverse range of studies that facilitate the sharing of knowledge and development of skills required for practising architecture within an interdisciplinary environment.

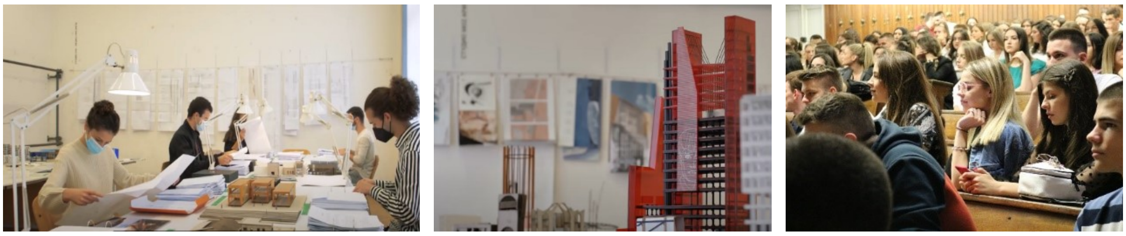

Figure 1. Faculty of Architecture University of Belgrade

The learning process is structured around the activities of the Studio Project, which comprises more than half of the active class capacity and serves as the basis of architectural education. This project-based curriculum emphasizes practical work on specific projects and is complemented by a range of theoretical and research courses. A significant part of these courses focuses on mathematical concepts and techniques, both classical and those based on the latest advancements in technology. By integrating mathematical courses into architectural studies, the Faculty is helping to ensure that future architects have the necessary skills to design buildings that are both aesthetically pleasing and functional, meeting the needs of society. The inclusion of mathematics in the curriculum is essential for producing well-rounded architects capable of meeting the challenges of the ever-evolving architectural profession. In addition, a quality mathematical education can increase the competitiveness of graduate architects in the job market, where companies and clients seek experts with a broad range of knowledge and skills, including mathematical proficiency, in today's rapidly evolving world.

We feature courses taught by the authors of this paper:

- Mathematics in architecture (a core course in the first year of undergraduate studies);
- Architectural geometry I/II (two core courses in the first year of undergraduate studies);
- Applied mathematics in the field of constructive systems (a core course in the first year of master studies in the programme Construction Engineering);

- Parametric modelling of architectural forms (an elective course in the first year of master studies available in all programmes);
- Integrated modelling of architectural form (an elective course in the first year of master studies available in all programmes);
- Building information modelling (BIM) I/II (two elective courses in the first year of master studies available in all programmes).

The Mathematics in architecture course introduces students to the foundational concepts of analytic and fractal geometry, providing them with the technical tools they need to understand the geometrical principles that underpin architectural design.

Architectural geometry courses focus on descriptive geometry and aim to develop students' logical thinking, spatial perception, and ability to imagine three-dimensional space. Students learn about the geometry of architectural forms and use this knowledge to define constructive elements for both exterior and interior design. They also learn various geometric-constructive methods for processing and representing 3D forms applicable in architecture, such as "orthogonal views" obtained for mutually parallel rays of perception and modern digital technology requirements.

The Applied mathematics in the field of constructive systems provides the necessary foundation for master's students to understand the complex mathematical principles of differential and integral calculus that are essential in solving extremal geometric problems, as well as determining the curvature of curves and surfaces and calculating the areas and volumes of irregular figures and solids.

Other courses listed above, related to digital modelling, programming, and other information technologies, enable students to create complex design more efficiently and accurately. These technologies also allow students to simulate and visualize their designs in a virtual environment, which can help to identify and solve problems before construction begins. In order to be able to use digital modelling and programming tools effectively, students need to have a solid understanding of mathematical principles. Hence, the integration of mathematics and digital technologies is crucial for the education of future architects, as it enables them to develop the skills and knowledge required to meet the demands of a rapidly changing profession. It is not uncommon to deviate from the planned curriculum on these courses in order to address specific, current issues or ongoing problematic situations that students are facing during their Studio project. In such cases, mathematics, supported by computer technology, can offer solutions, enabling students to apply a flexible approach and think creatively in problem-solving.

Playing and learning with and in IFS

After students acquire a solid foundation of theoretical mathematics using standard teaching methods, they often struggle to see the practical application of this knowledge in their future professional activities. Many students fail to recognize the potential benefits of mathematics and struggle to understand how to integrate it into their work. In order to bridge this gap between theory and practice, it is crucial to incorporate experimental work or functional research into the curriculum after each thematic unit. By doing so, students are better able to understand the practical implications of mathematical concepts and can more effectively apply their knowledge in real-world situations. To this end, the active learning method implemented on IFS during the Mathematics in architecture course, serves as a prime example of how to incorporate hands-on, practical experiences into the curriculum.

Fractals are complex geometric patterns that are self-similar at different scales. They are often used in architecture, design, and art because of their ability to create visually interesting structures. IFS are one of the techniques used to create fractals. An IFS is a set of mathematical functions that are iteratively applied to a set of points, creating a self-similar pattern. The process can be repeated many times, with different parameters and initial conditions, resulting in a wide variety of fractal shapes. Fractal geometry is useful in architecture for creating buildings with irregular shapes and patterns, as well as for optimizing structural efficiency. For more theoretical details about IFS, see [1] and [2].

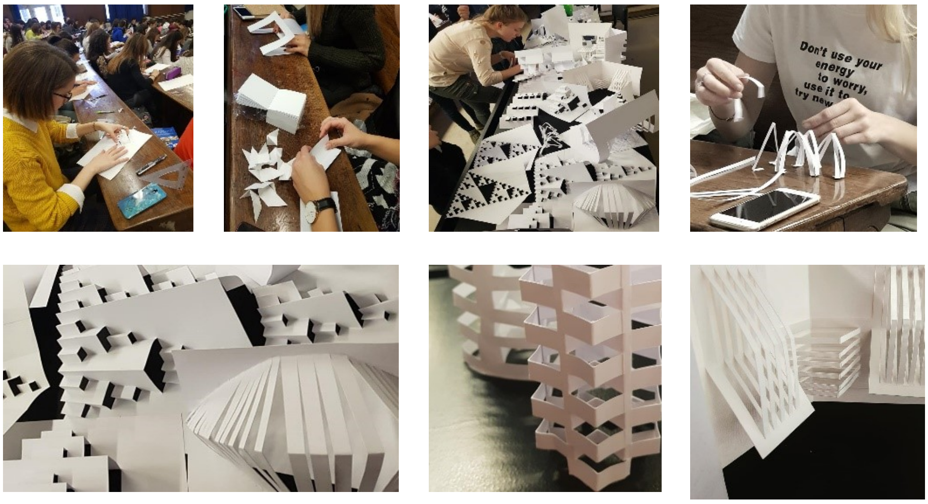

Figure 2. Initial exploring during class time

After learning the theory related to IFS, students are assigned to create an IFS fractal of their own design. This project aims to develop the students' skills in constructing mathematical models and exploring the creative possibilities of fractals in architecture and design. To be precise, we conduct a workshop in which students create several analog fractal models using paper or other inexpensive and easily available materials. The initial goal is for each student to explore the creation of a sequence of different models during class time. This is achieved through a process of rough selection followed by careful refinement, using different initiators and various affine transformations (translation, rotation, scaling) that are iteratively repeated the desired number of times (see Figure 2.).

At the end of the class, we group the works based on their quality in terms of creativity, originality, and architectural potential. We discuss with the students and give them the

opportunity to act as evaluators and express which models they would choose as the most high-quality and provide a justification for their selections. This collaborative approach not only fosters a sense of community among the students, but also allows them to reflect on their own design choices and learn from the successes and challenges of others. Through comparative analysis, they simultaneously consider ways to improve their own models. This phase of the process is especially enjoyable for the students.

Afterward, the students have one week to complete the task, i.e. to generate their final model. They must first decide on the initial form, select the materials to be used, choose the transformations to be applied, and determine the number of iterations. In this way, they are encouraged to experiment with various forms and materials, and to consider the potential for architectural or design applications of their fractal creations. At the end, once completed, students are required to photograph their IFS fractal and submit the photograph that highlights the architectural or design potential to an online forum created specifically for the course. Through this forum, each student has the opportunity to showcase their work to their colleagues, to positively react to the results they consider most successful and receive valuable feedback on their own creations. As evaluators, teachers also benefit from the forum by gaining a more comprehensive understanding of the outcomes considering that there are a large number of enrolled students (over 300). By observing the students' work, we can objectively evaluate their projects and assess their understanding of the underlying concepts of IFS fractals. Additionally, the forum enables us to identify any areas of the course that require further clarification or improvement, and to provide timely feedback to the students. Overall, the online forum serves as a valuable tool for both the students and instructors, facilitating a collaborative and dynamic learning environment. Figure 3 shows the most successful results of this workshop and serves as a prime example of a picture speaking a thousand words.

Conclusions

Expanding on the recommendations for the mathematical education of architects, our observations and findings highlight the crucial role of practical application in reinforcing theoretical concepts. It is important to create an environment where mathematical concepts are not just taught as abstract theories, but are also reinforced through concrete and visually tangible applications that have clear architectural potential. By providing opportunities for hands-on experimentation and active learning, students can not only deepen their understanding of mathematical principles but also appreciate how those principles can be practically applied in their future careers as architects. The ability to visualize and see the potential applications of mathematical concepts in architecture can also spark creativity and inspire innovative design solutions. Therefore, a curriculum that emphasizes both theoretical and practical aspects of mathematics can provide students with a well-rounded education that prepares them for the challenges and demands of the architectural profession. Regular refinement and updating of the curriculum to keep up with the changing technological landscape is also essential to ensure that future architects are equipped with the necessary skills and knowledge to succeed in their careers.

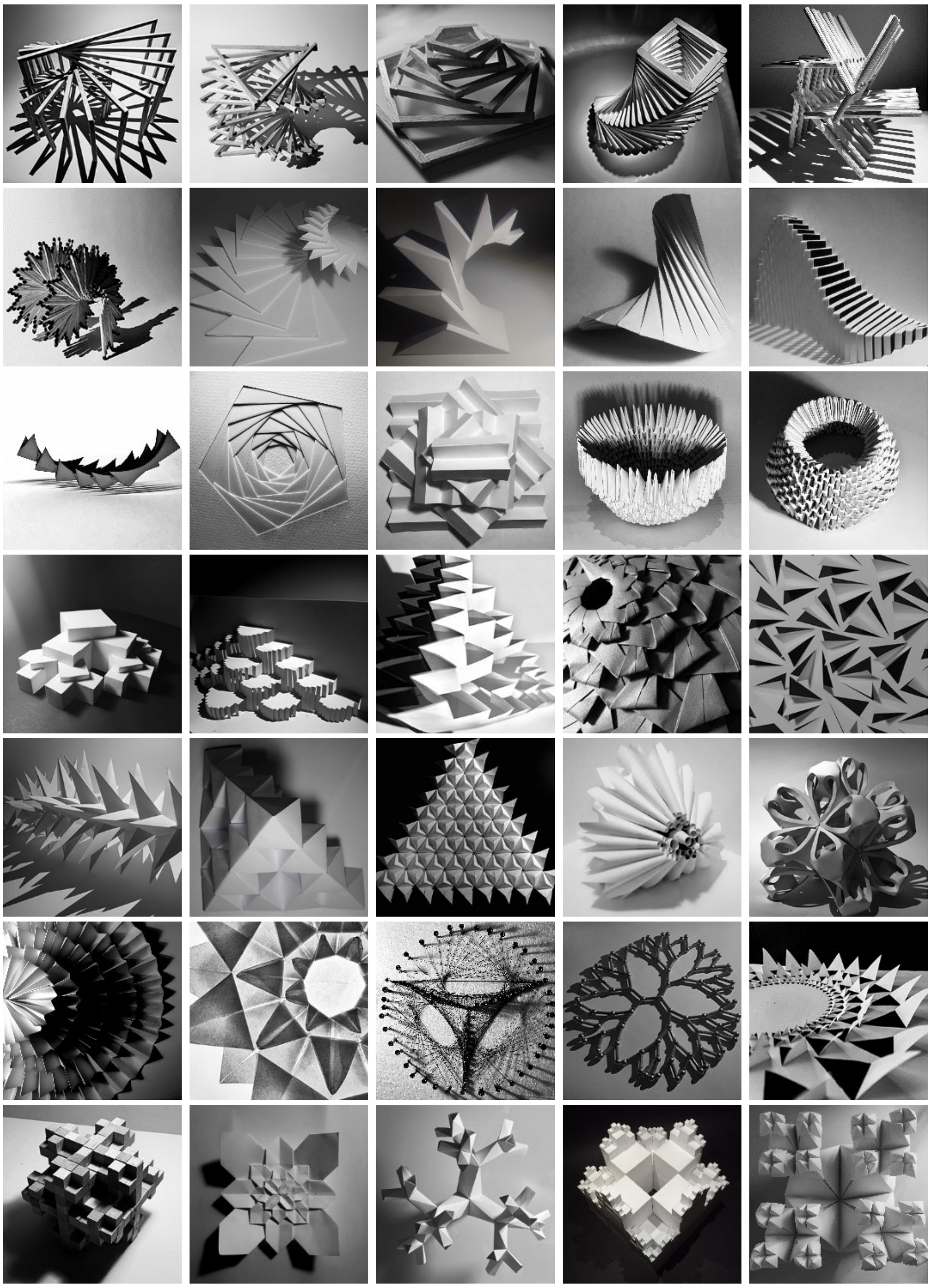

Figure 3. Workshop results – IFS models

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