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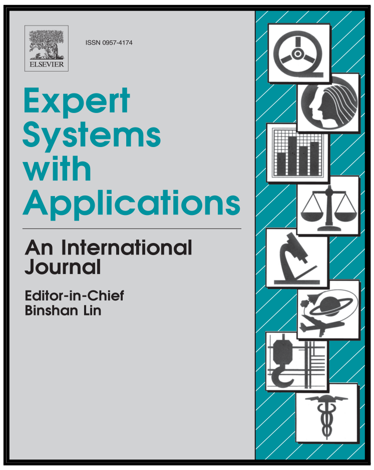


Novel approach to group multi-criteria decision making based on interval rough numbers: Hybrid DEMATEL-ANP-MAIRCA model

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**Highlights**

- Interval rough number is introduced to deal with the vagueness in decision-making
- A novel DEMATEL-ANP model based oninterval rough numbers
- Application of a new multi-criteria technique called MARICA
- An interval rough number based on MARICA is proposed to evaluate the alternatives
- Multi-criteria techniques were compared based on interval rough and fuzzy approaches.

# Novel approach to group multi-criteria decision making based on interval rough numbers: Hybrid DEMATEL-ANP-MAIRCA model

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*Abstract*
This paper presents a novel approach for treating uncertainty in the multi-criteria decision making process by introducing interval rough numbers (IRN). The IRN approach enables decision making using only the internal knowledge incorporated in the data provided by the decision maker. A hybrid multi-criteria model was developed based on IRN, and demonstrated using the example of the bidder selection process in the state administration public procurement procedure. The first segment of the hybrid model deals with the rough interval DEMATEL-ANP (IR'DANP) model, which enables more objective expert evaluation of criteria in a subjective environment than the traditional/crisp approach. In the second segment, the evaluation is enabled by applying the new rough interval MAIRCA method, which introduces mathematical tools and shows high stability concerning changes in the nature and characteristics of the criteria. The results of the hybrid IR'DANP-MAIRCA model were analyzed using 36 scenarios of sensitivity analysis, which showed high stability of the results. The results of the interval rough method were compared with the fuzzy extensions of the TOPSIS, VIKOR, MABAC, TODIM, ELECTRE I and DEMATEL-ANP models.
*Key words*: interval rough numbers; DEMATEL; ANP; MAIRCA; public procurements.

## 1. Introduction

The process of selecting alternatives in an MCDM problem assumes that the psychology and behavior of the decision makers (DM) will be completely rational (Fan et al., 2013). However, in reality, experts with different backgrounds and levels of knowledge use linguistic terms to represent their evaluation and also their preferences while solving qualitative group decision-making problems (Xu and Wang, 2016). In general, data and information found in the judgment of the experts are subjective as well as inherently non-numeric, and this gives rise to uncertainty and impreciseness with non-probabilistic characteristics (Martinez et al., 2007). Hence, various approaches can be used to enable more realistic presentation of the decision attribute values: interval numbers (Zeshui and Qingli, 2003; Shuping, 2009), fuzzy sets (Zadeh, 1965; Pamučar and Ćirović, 2015), rough numbers (Song et al., 2014; Zhu et al., 2015), grey theory (Kuang et al., 2015; Arce et al., 2015), Z numbers (Kang et al., 2012; Azadeh and Kokabi, 2016), and others. These approaches are most appropriate for presenting uncertainties related to describing qualitative criteria using linguistic scales, defining indicators for qualitative criteria, and for the reliability of expert evaluations. The basic idea of applying algorithms in the decision making process supported by the interval approach (interval numbers, grey theory and so on) implies that interval numbers will be used for presenting the attribute values. However, it is very difficult to define the limits of the interval numbers since they are all based on experience, intuition and the subjective perception of the decision maker.

To deal with uncertainties and to determine the values of qualitative attributes, the majority of authors use fuzzy sets (Zadeh, 1965) or various extensions of fuzzy theory such as: interval-valued fuzzy sets (Vahdani et al., 2013; Sizong and Tao, 2016; Zywica, 2016), intuitionistic fuzzy sets (Atanassov, 1986; Ngan, 2017), interval intuitionistic fuzzy sets (Nayagama, 2016; Nguyen, 2016), hesistant fuzzy sets (Wang et al., 2015; Ngan, 2017),

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and the like. Fuzzy sets are a very powerful and commonly used tool for dealing with imprecision. However, subjectivity when selecting an appropriate membership function for fuzzy sets can affect the final decision and so particular care needs to be paid to it (Qazi et al., 2016; Wang et al., 2016).

In addition to fuzzy theory, rough set theory, originally introduced by Pawlak (1982), is another suitable tool for treating imprecision. In recent years, rough set theory has been successfully implemented in various fields of human activities. It can be said that its application is adequate and usually irreplaceable when handling uncertainty and inaccuracy analyses. Knowing the advantages of rough set theory (Pawlak 1991), the application of rough sets is fully justified in today's modern practice in the decision-making process when it includes imprecision in the data.

The purpose of the fuzzy tehnique in the decision making process is to enable the transformation of crisp numbers into fuzzy numbers that show uncertainties in real world systems using the membership function. As opposed to fuzzy sets theory that requires a subjective approach in determining partial functions and fuzzy set boundaries, rough set theory determines set boundaries based on real values and depends on the degree of certainty of the decision maker. Since rough set theory deals solely with internal knowledge, i.e. operational data, there is no need to rely on assumption models. In other words, when applying rough sets, only the structure of the given data is used instead of various additional/external parameters (Yang et al., 2016). Duntsch and Gediga (1997) believe that the logic of rough set theory is based solely on data that speak for themselves. When dealing with rough sets, the measurement of uncertainty is based on the vagueness already contained in the data (Xu et al., 2016b). In this way, the objective indicators contained in the data can be determined. In addition, rough set theory is suitable for application on sets characterized by irrelevant data where use of statistical methods does not seem appropriate (Pawlak 1991, 1993; Zhang et al., 2016).

## 2. Literature revew

From its beginnings until today, rough set theory has evolved by solving numerous soft computing problems (Khoo and Zhai, 2001; Li et al., 2009; Zhai et al., 2010; Nauman et al., 2016; Liang et al., 2017), as well as applying rough numbers in a QFD matrix (Zhai et al., 2008), evaluating the requirements of industrial product service system users by applying rough numbers (Song et al., 2013a), design concept evaluation (Zhu et al., 2015), product design evaluation (Tiwari et al., 2016; Hesam et al., 2016) and so on. Since this paper deals with the application of interval rough numbers in the multi-criteria decision making process, studies referring to the modification of multi-criteria decision making (MCDM) models by applying rough numbers and their extensions are presented below. Special attention is given to literature that deals with the bidder selection process in the public procurement procedure since this is the case study used to demonstrate the interval rough MCDM model.

Some papers deal with the application of rough numbers in multi-criteria models that utilize the rough AHP method either on its own (Sugihara et al., 1999; Xie et al., 2008; Li et al., 2009; Kang et al., 2016) or as a hybrid model in combination with other multi-criteria techniques: AHP-TOPSIS (Aydogan, 2011; Song, 2014), AHP-VIKOR (Guo and Zhang, 2008; Ağirgün, 2012; Zhu et al., 2015) and AHP-MABAC (Roy et al., 2016). However, only a few studies deal with the application of rough sets (Pawlak 1991, 1993) and rough numbers (Hesam et al., 2016) in the MCDM process even though they show considerable advantages. The authors have also found very few papers dealing with the application of interval rough numbers in MCDM. Wang et al. (2011) applied interval rough numbers to deal with imprecision when determining the weight coefficients of decision attributes by introducing an interval rough operator for IRN aggregation. IRN was also applied to develop a hybrid QFD model (Zheng et al., 2016).

On the other hand, analysis of the literature that deals with the application of MCDM in the public procurement procedure (Table 1) has shown that the crisp and fuzzy approaches are the most frequently applied MCDM techniques.

Table 1. Application of MCDM techniques in the bidder evaluation process -public procurement procedure

| Technique | Fuzzy (literature) | Traditional-crisp (literature) |
|---|---|---|
| AHP/ANP | Kahraman et al. (2003); Dobi et al. (2010); Amid, Ghodsypour, and O'Brien (2011); Labib (2011); Costantino et al. (2011); Han and Wang, (2016); Nazari et al. (2016); Sameh et al. (2016) | Topcu (2004); Levary (2008); Bhattacharya et al. (2010); Sipahi and Esen (2010); Chan and Chan (2010); Ho et al. (2011); Mafakheri et al. (2011); Ishizaka et al. (2012); Yu et al. (2012); Veselinović (2014); ); Chua et al., (2015); Mimović and Krstić (2016) |
| TOPSIS | Crispim and De Sousa (2010); Zhao | Adil et al. (2014); Ekbatania and Cats |

| Technique | Fuzzy (literature) | Traditional-crisp (literature) |
|---|---|---|
| | and Guo (2014); Bobar et al. (2015); Liu et al., (2016) | (2015) |
| ELECTRE I | | Vahdani et al. (2010); Liu and Zhang (2011); Zak (2015) |
| Hybrid model ANP-TOPSIS | Kuo et al. (2015) | Lin et al. (2011); Buyukozkan and Cifci (2012) |
| The best worst method | Rezaei et al. (2016) | |
| Max–min method | Sen et al. (2010) | |
| DEA (Data Envelopment Analysis) | Ji et al., 2016 | Falagario et al. (2012) |

Based on the literature survey (Table 1), it can be noticed that authors have given much attention to both the development and application of various multi-criteria techniques in public procurements. It is worth mentioning that public procurements are considered to be an important segment of the market, having in mind that the cost of public procurements in the European Union amounts to nearly 19% of the GDP (European Commission, 2012), and 15% of the global GDP (Bajari and Lewis, 2011). Hence, there is a need to take full control over the public procurement procedure when it comes to legality and spending the available funds for designated purposes. Multi-criteria techniques are considered to be one of the tools that can provide objectivity, efficiency and regularity in the public procurement process.

The literature survey shows that traditional multi-criteria techniques and their fuzzy extensions are most commonly used. Therefore, using other approaches such as IRN and comparing it with both the traditional and fuzzy approaches is a logical step towards upgrading the methodology required for evaluating bidders in public procurement procedures. In addition, by analyzing the data from Table 1, we can see that the most commonly applied multi-criteria techniques are AHP/ANP, TOPSIS and ELECTRE I, as well as hybrid models formed by combining them. Therefore, this paper compares the proposed MCDM model with the crisp and fuzzy approaches of these multi-criteria techniques.

The DEMATEL (Decision-Making Trial and Evaluation Laboratory) method (Gabus and Fontela, 1976), developed in the Geneva Research Centre of the Battelle Memorial Institute, is one of the methods that can be used to model causal dependencies among criteria. The DEMATEL method is able to visualize the complex cause and effect relationships in an understandable manner (Büyüközkan & Çifçi, 2012). Dalalah et al. (2011) used the DEMATEL method for estimating the criteria weights for the supplier selection problem, Chou et al. (2012) combined fuzzy DEMATEL and fuzzy AHP to analyze human resources evaluation criteria, Lin and Wu (2008) applied fuzzy DEMATEL to the R&D project selection problem. Tseng (2009) developed a grey-fuzzy DEMATEL method for the real estate quality agent ranking problem. Lin (2013) evaluated green supply chain practices using the fuzzy DEMATEL method. In addition to the above-mentioned applications of the DEMATEL method, the literature also deals with numerous modifications that use spatial fuzzy models (Pamučar and Ćirović, 2015; Jin et al., 2016; Gigović et al., 2017), D number theory (Zhou et al., 2017), and grey theory (Su et al., 2016; Zhong and Chen, 2016; Liang et al., 2016; Shao et al., 2016). Based on the literature survey and the knowledge of the authors, no modification of the DEMATEL method applying interval rough numbers (IR-DEMATEL) has been analyzed in the literature so far.

Numerous examples of DEMATEL-ANP models for determining weight coefficients have been analyzed by various authors (Yang et al., 2013; Gong et al., 2016; Govindan et al., 2014; Kuo et al., 2015; Pourahmad et al., 2015; Shen and Tzeng, 2016; Gigovic et al., 2017). However, only Buyukozkan and Cifci (2012) have analyzed the application of the crisp DEMATEL-ANP model in the public procurement process. In addition, since a DEMATEL-ANP model based on interval and/or standard rough numbers has not been developed yet, this paper describes an original approach. The IR'DANP model integrates a new method within the MCDM - MAIRCA method (Pamučar et al., 2014) which was adjusted for application in the interval rough environment. The MAIRCA method is a simple mathematical tool with a high degree of stability related to changes in both the nature and character of the criteria (Gigovic et al., 2016). According to our knowledge, neither the public procurement process nor MCDM recognize a hybrid model able to analyze interdependence among the criteria, evaluate alternative solutions or treat imprecision by applying interval rough numbers.

This paper has several objectives. The first is to upgrade the methodology for treating uncertainty in the group multi-criteria decision making process. The second objective is to affirm the IRN idea through detailed presentation of the arithmetic operations typical for MCDM using IRN. The third objective is to encourage other authors to start with the widespread application of IRN in MCDM, since the advantages of IRN, as presented in this paper, offer reasonable motive for widespread application. And finally, the fourth objective of this paper is to bridge the gap identified in the methodology for the bidder evaluation process in the public procurement procedure by applying a novel approach to treating uncertainties based on IRN.

One of the contributions developed in this paper is the introduction of the IR'DANP-MAIRCA model that provides more objective expert evaluation of criteria in a subjective environment. Another significant contribution is the introduction of the novel IR-DEMATEL, IR-ANP and IR-MAIRCA models developed by various authors for the purpose of upgrading MCDM techniques. These models enable the evaluation of alternative solutions despite dilemmas in the decision making process and lack of quantitative information.

The remainder of this paper is structured as follows: Section 3 gives a brief idea of interval rough numbers using mathematical equations. Section 4 proposes an algorithm for the hybrid IRD'ANP-MAIRCA model which is demonstrated using the real example of bidder evaluation in the public procurement procedure as described in Section 5. Section 6 presents a discussion of the IRD'ANP-MAIRCA model results. The discussion of the results is presented by means of a sensitivity analysis and comparison of the results with fuzzy and rough extensions of the TOPSIS, ELECTRE I, MABAC and VIKOR methods. Finally, Section 7 presents the conclusions, highlighting directions for further research.

## 3. Interval rough numbers

Assume that $U$ is the universe containing all the objects registered in an information table. Assume that there is a set of $k$ classes representing the DM preferences $R=(J_1,J_2,...,J_k)$ provided that they belong to a row which satisfies the condition $J_1<J_2<...<J_k$ and another set of $k$ classes that also represent the DM preferences $R^*=(I_1,I_2,...,I_k)$. Assume that all objects are defined in a universe and related to the DM preferences. In $R^*$ every class of objects is represented by interval $I_i=\{I_{li},I_{ui}\}$, provided that $I_{li}\le I_{ui}$ ($1\le i\le m$), and $I_{li},I_{ui}\in R$ are satisfied. Then, $I_{li}$ denotes the lower interval limit, while $I_{ui}$ denotes the upper interval limit of $i$ class. If both class limits (lower and upper limits) are presented so that $I^*_{l1}<I^*_{l2}<,...,<I^*_{lj},I^*_{u1}<I^*_{u2}<,...,<I^*_{uk}$ $(1\le j,k\le m)$ are satisfied respectively, then two new sets containing the lower class $R^*_l=(I^*_{l1},I^*_{l2},...,I^*_{lj})$ and upper class $R^*_u=(I^*_{u1},I^*_{u2},...,I^*_{uk})$ can be defined respectively. If such is the case, then for any class $I^*_{li}\in R$ $(1\le i\le j)$ and $I^*_{ui}\in R$ $(1\le i\le k)$ the lower approximation of $I^*_{li}$ and $I^*_{ui}$ can be defined as follows (Wang et al., 2011):

$$\underline{Apr}(I^*_{li})=\cup\left\{Y\in U/R^*_l(Y)\le I^*_{li}\right\} \tag{1}$$

$$\underline{Apr}(I^*_{ui})=\cup\left\{Y\in U/R^*_u(Y)\le I^*_{ui}\right\} \tag{2}$$

The above-mentioned approximations of $I^*_{li}$ and $I^*_{ui}$ are defined by applying the following equation

$$\overline{Apr}(I^*_{li})=\cup\left\{Y\in U/R^*_l(Y)\ge I^*_{li}\right\} \tag{3}$$

$$\overline{Apr}(I^*_{ui})=\cup\left\{Y\in U/R^*_u(Y)\ge I^*_{ui}\right\} \tag{4}$$

Both object classes (upper and lower classes $I^*_{li}$ and $I^*_{ui}$) are defined by their lower limits $\underline{Lim}(I^*_{li})$ and $\underline{Lim}(I^*_{ui})$ and upper limits $\overline{Lim}(I^*_{li})$ and $\overline{Lim}(I^*_{ui})$, respectively

$$\underline{Lim}(I^*_{li})=\frac{1}{M_L}\sum R^*_l(Y)\big|Y\in\underline{Apr}(I^*_{li}) \tag{5}$$

$$\underline{Lim}(I^*_{ui})=\frac{1}{M^*_L}\sum R^*_u(Y)\big|Y\in\underline{Apr}(I^*_{ui}) \tag{6}$$

where $M_L$ and $M^*_L$ denote the number of objects contained in lower approximations $I^*_{li}$ and $I^*_{ui}$, respectively. The upper limits $\overline{Lim}(I^*_{li})$ and $\overline{Lim}(I^*_{ui})$ are defined by equations (7) and (8)

$$\overline{Lim}(I^*_{li})=\frac{1}{M_U}\sum R^*_l(Y)\big|Y\in\overline{Apr}(I^*_{li}) \tag{7}$$

$$\overline{Lim}(I^*_{ui})=\frac{1}{M^*_U}\sum R^*_u(Y)\big|Y\in\overline{Apr}(I^*_{ui}) \tag{8}$$

where $M_U$ and $M^*_U$ denote the number of objects contained in upper approximations $I^*_{li}$ and $I^*_{ui}$, respectively.

For the lower class of objects, the rough boundary interval from $I^*_{li}$ is represented as $RB(I^*_{li})$ and denotes the interval between the lower and upper limits:

$$RB(I_{li}^*) = \overline{Lim}(I_{li}^*) - \underline{Lim}(I_{li}^*) \tag{9}$$

While for the upper object class, the rough boundary interval $I_{ui}^*$ is obtained based on the following equation

$$RB(I_{ui}^*) = \overline{Lim}(I_{ui}^*) - \underline{Lim}(I_{ui}^*) \tag{10}$$

Then the uncertain class of objects $I_{li}^*$ and $I_{ui}^*$ can be expressed using their lower and upper limits

$$RN(I_{li}^*) = \left[\overline{Lim}(I_{li}^*), \underline{Lim}(I_{li}^*)\right] \tag{11}$$

$$RN(I_{ui}^*) = \left[\overline{Lim}(I_{ui}^*), \underline{Lim}(I_{ui}^*)\right] \tag{12}$$

It can be seen that every class of objects is defined by its lower and upper limits which create an interval rough number that can be defined as:

$$IRN(I_i^*) = \left[RN(I_{li}^*), RN(I_{ui}^*)\right] \tag{13}$$

The procedure for defining IRN will be explained while determining the weight coefficient of criterion $w_i$. The criteria were evaluated by four experts. These experts evaluated the criteria using a scale ranging from 1 to 5: 1-very low, 2-low, 3-moderate, 4-high, and 5-very high. The expert evaluations are shown in Table 2.

Table 2. Expert evaluations of criterion $w_i$

| Criterion | Experts | | | |
|---|---|---|---|---|
| | E1 | E2 | E3 | E4 |
| $w_i$ | (2;3) | (3;4) | (4;5) | (5;5) |

The expert evaluations given in Table 2 are shown in the form of ordered pairs ($a_i$;$b_i$), where $a_i$ and $b_i$ denote values assigned to the criteria based on a 1-5 scale. If an expert cannot decide on only one value from this scale then both values are considered (E1, E2 and E3). The above-mentioned example shows that only expert E4 has no dilemma since he decided on a unique value from the scale.

These uncertainties can be represented by trapezoidal fuzzy numbers in the form $A$=($a_1$, $a_2$, $a_3$, $a_4$), where $a_2$ and $a_3$ denote values in which the membership function can reach its maximum value, while $a_1$ and $a_4$ denote the left and right boundaries of the fuzzy set, respectively. In the above-mentioned example (Table 2) we obtain four trapezoidal fuzzy numbers $A$(E1)=(1,2,3,4), $A$(E2)=(2,3,4,5), $A$(E3)=(3,4,5,5) and $A$(E4)=(4,5,5,5). A graphic presentation of the trapezoidal fuzzy numbers is shown in Figure 1, where the darker shade denotes values in which the membership function can reach its maximum value ($a_2$ and $a_3$), while the lighter shade denotes elements of the set that more or less belong to fuzzy set ($a_1$ and $a_4$).

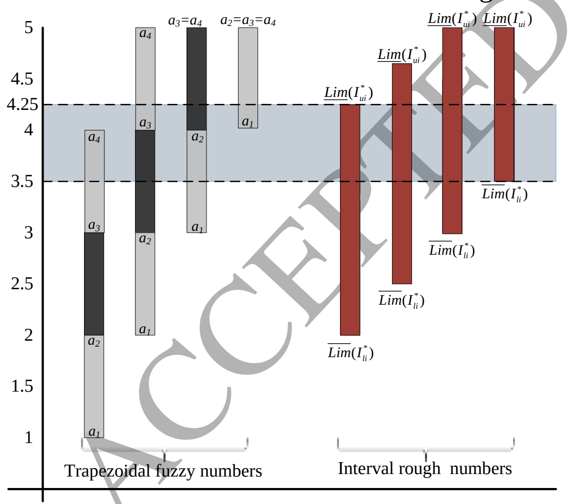


Figure 1. Evaluation of criteria – an interval rough and fuzzy evaluation

In addition to the fuzzy approach, these imprecisions can also be represented by interval rough numbers. Since in the above-mentioned equations (1) through (12) an IRN is composed of two rough sequences, the following two classes of objects $w_i$ and $w'_i$ can be defined: $w_i = \{2;3;4;5\}$ and $w_i' = \{3;4;5;5\}$. By applying equations (1) through (8), rough sequences (11) and (12) can be established for every object class. For the first object class, we obtain:

$\underline{Lim}(2) = 2$, $\overline{Lim}(2) = \frac{1}{4}(2+3+4+5) = 3.5$; $RN(2) = [2, 3.5]$

$\underline{Lim}(3)=\frac{1}{2}(2+3)=2.5$, $\overline{Lim}(3)=\frac{1}{3}(3+4+5)=4$; $RN(3)=[2.5,4]$

$\underline{Lim}(4)=\frac{1}{3}(2+3+4)=3$, $\overline{Lim}(4)=\frac{1}{2}(4+5)=4.5$; $RN(4)=[3,4.5]$

$\underline{Lim}(5)=\frac{1}{4}(2+3+4+5)=3.5$, $\overline{Lim}(5)=5$; $RN(5)=[3.5,5]$

For the second object class we obtain:

$\underline{Lim}(3)=3$, $\overline{Lim}(3)=\frac{1}{4}(3+4+5+5)=4.25$; $RN(3)=[3,4.25]$

$\underline{Lim}(4)=\frac{1}{2}(3+4)=3.5$, $\overline{Lim}(4)=\frac{1}{3}(4+5+5)=4.67$; $RN(4)=[3.5,4.67]$

$\underline{Lim}(5)=\frac{1}{4}(3+4+5+5)=4.25$, $\overline{Lim}(5)=5$; $RN(5)=[4.25,5]$

Based on the rough sequences, the following interval rough numbers are obtained: $IRN(E1)=\left([2,3.5],[3,4.25]\right)$, $IRN(E2)=\left([2.5,4],[3.5,4.67]\right)$, $IRN(E3)=\left([3,4.5],[4.25,5]\right)$ and $IRN(E4)=\left([3.5,5],[4.25,5]\right)$.

Based on rational judgment without introducing rough and fuzzy sets, it can be concluded that values of criterion $w_i$ should range between 3.5 and 4.25. These values are obtained based on the geometric mean of classes $w_i=\{2;3;4;5\}$ and $w_i^{'}=\{3;4;5;5\}$. The rational (expected) values 3.5 and 4.25 are represented by a broken line, Figure 1. It can be seen that the expected values (3.5 and 4.25) are completely noticeable within all IRN, as shown in Figure 1. On the other hand, the fuzzy numbers only partially include the expected values. Therefore the membership function of fuzzy numbers *A*(E2) and *A*(E3) partially includes the expected values, while fuzzy numbers *A*(E1) and *A*(E4) include the value of 0.5. On the other hand, all IRNs completely include the expected values (3.5 and 4.25).

The interval rough numbers are characterized by specific arithmetic operations that differ from those dealing with typical rough numbers. The arithmetic operations between two interval rough numbers $IRN(A)=\left([a_1,a_2],[a_3,a_4]\right)$ and $IRN(B)=\left([b_1,b_2],[b_3,b_4]\right)$ are carried out using the following expressions (14), (15), (16), (17) and (18) (Wang et al., 2011):

(1) Addition of interval rough numbers"+"

$$IRN(A)+IRN(B)=\left([a_1,a_2],[a_3,a_4]\right)+\left([b_1,b_2],[b_3,b_4]\right)=\left([a_1+b_1,a_2+b_2],[a_3+b_3,a_4+b_4]\right) \tag{14}$$

(2) Substraction of interval rough numbers"-"

$$IRN(A)-IRN(B)=\left([a_1,a_2],[a_3,a_4]\right)-\left([b_1,b_2],[b_3,b_4]\right)=\left([a_1-b_4,a_2-b_3],[a_3-b_2,a_4-b_1]\right) \tag{15}$$

(3) Multiplication of interval rough numbers"×"

$$IRN(A)\times IRN(B)=\left([a_1,a_2],[a_3,a_4]\right)\times\left([b_1,b_2],[b_3,b_4]\right)=\left([a_1\times b_1,a_2\times b_2],[a_3\times b_3,a_4\times b_4]\right) \tag{16}$$

(4) Division of interval rough numbers"/"

$$IRN(A)/IRN(B)=\left([a_1,a_2],[a_3,a_4]\right)/\left([b_1,b_2],[b_3,b_4]\right)=\left([a_1/b_4,a_2/b_3],[a_3/b_2,a_4/b_1]\right) \tag{17}$$

(5) Scalar multiplication of interval rough numbers, where $k>0$

$$k\times IRN(A)=k\times\left([a_1,a_2],[a_3,a_4]\right)=\left([k\times a_1,k\times a_2],[k\times a_3,k\times a_4]\right) \tag{18}$$

Based on the rules for comparing standard rough numbers (Zhai et al., 2008), the authors of this paper determined rules for ranking IRN. Any two interval rough numbers $IRN(\alpha)=\left(\left[\alpha^L,\alpha^U\right],\left[\alpha^{'L},\alpha^{'U}\right]\right)$ and $IRN(\beta)=\left(\left[\beta^L,\beta^U\right],\left[\beta^{'L},\beta^{'U}\right]\right)$ are ranked by applying the following rules:

(1) If an interval rough number is not strictly bounded by another interval, then:

- (a) If condition { $\alpha^{'U}>\beta^{'U}$ and $\alpha^L\geq\beta^L$ } or { $\alpha^{'U}\geq\beta^{'U}$ and $\alpha^L<\beta^L$ } is satisfied, then $IRN(\alpha)>IRN(\beta)$, Figure 2a.
- (b) If condition { $\alpha^{'U}=\beta^{'U}$ and $\alpha^L=\beta^L$ } is satisfied, then $IRN(\alpha)=IRN(\beta)$, Figure 2b.

(2) If interval rough numbers $IRN(\alpha)$ and $IRN(\beta)$ are strictly bounded, then the intersection points $I(\alpha)$ and $I(\beta)$ of interval rough numbers $IRN(\alpha)$ and $IRN(\beta)$ are determined. If condition $\beta'^{U} < \alpha'^{U}$ and $\beta^{L} > \alpha^{L}$ is satisfied, then

(a) If condition $I(\alpha) \le I(\beta)$ is satisfied, then $IRN(\alpha) < IRN(\beta)$, Figures 2c and 2d.

(b) If condition $I(\alpha) > I(\beta)$ is satisfied, then $IRN(\alpha) > IRN(\beta)$, Figure 2e.

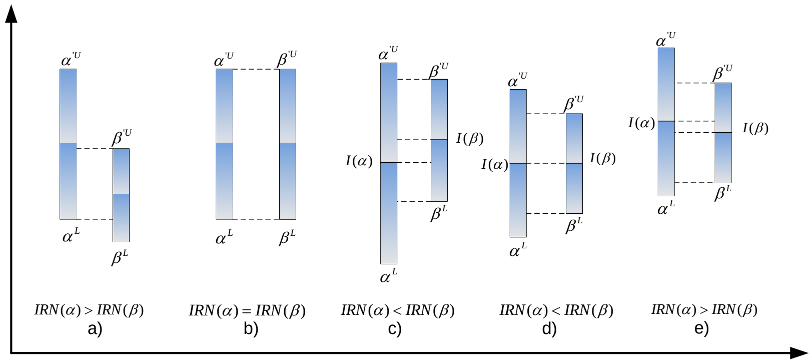


Figure 2. Ranking interval rough numbers

Intersection points of the interval rough numbers will be obtained as follows:

$$\mu_\alpha = \frac{RB(\alpha_{ui})}{RB(\alpha_{ui}) + RB(\alpha_{li})};\quad RB(\alpha_{ui}) = \alpha'^{U} - \alpha'^{L};\quad RB(\alpha_{li}) = \alpha^{U} - \alpha^{L} \tag{19}$$

$$\mu_\beta = \frac{RB(\beta_{ui})}{RB(\beta_{ui}) + RB(\beta_{li})};\quad RB(\beta_{ui}) = \beta'^{U} - \beta'^{L};\quad RB(\beta_{li}) = \beta^{U} - \beta^{L} \tag{20}$$

$$I(\alpha) = \mu_\alpha \cdot \alpha^{L} + (1 - \mu_\alpha) \cdot \alpha'^{U} \tag{21}$$

$$I(\beta) = \mu_\beta \cdot \beta^{L} + (1 - \mu_\beta) \cdot \beta'^{U} \tag{22}$$

Similar rules can be applied provided that $\alpha'^{U} < \beta'^{U}$ and $\alpha^{L} > \beta^{L}$.

## 4. Hybrid IRD'ANP-MAIRCA model

This paper presents a novel approach to the application of interval rough numbers in the group decision making process by introducing the hybrid IR D'ANP-MAIRCA model, Figure 3.

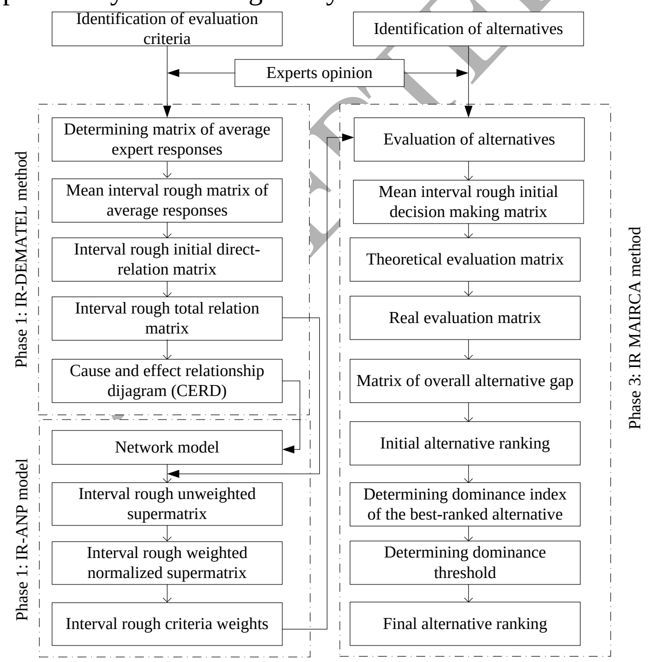


Figure 3. Framework of the proposed model

Interval rough numbers are used to deal with uncertainty in the group decision making process. Phase 1 includes the expert evaluation of criteria by applying the IR DEMATEL model, which results in the creation of input data required for the IR ANP model. The output data from the IR DEMATEL model are further processed using the algorithm for the IR ANP model. The output data from the IR D'ANP model are used to obtain the interval rough weight coefficients of the criteria. The hybrid IR D'ANP model, which is the subject-matter of this paper, represents a novel approach for dealing with uncertainty based on IRNs. For defining the final rank of alternatives, the IR MAIRCA method is used. This method was developed in the Research Centre of the Logistics Department, University of Defense in Belgrade (Pamučar et al., 2014). The following three sections deal with the algorithms for the IR D'ANP-MAIRCA model.

***4.1. The IR-DEMATEL method***

The DEMATEL method is a comprehensive method used in both the design and analysis of structural method characterized by the causal relations between complex factors (Gabus and Fontela, 1976). The results obtained from this method are the total direct and indirect effects of each factor on the other factors and vice versa. The DEMATEL method is used to identify the dependent factors and degree of dependence between them. The method is based on graph theory, which enables visual planning and problem solving so that all relevant factors can be classified into causal and consequential factors, for better understanding of their interrelations. This method makes it possible to better understand the complex structure of a problem and define the relations between factors, as well as the relations between the level of the structure and strength of influence of a factor (Gigović et al., 2017).

For the purpose of accepting the subjectivity in the collective decision making process, this paper modifies the DEMATEL method by applying interval rough numbers. The application of interval rough numbers eliminates the necessity for additional information for defining uncertain number intervals. In such a way, the quality of the existing data in the collective decision making process can be retained, as well as the experts' perception, which is expressed through the aggregation matrix. The text below shows the steps governing the IR-DEMATEL method, which was used in the group decision making process.

*Step 1. Analysis of factors by experts*. Assuming that there are $m$ experts in the research and $n$ observed factors (criteria), each expert should determine the degree to which criterion $i$ affects criterion $j$. Comparative analysis of the $i^{th}$ and $j^{th}$ criteria pairwise by $k$ expert is denoted as $x_{ij}^{e}$, where: $i=1,...,n;\ j=1,...,n$. The value of each $x_{ij}^{e}$ pair is an integer, where: 0 – no influence; 1 – low influence; 2 – medium influence; 3 – high influence; 4 – very high influence. The judgment of $e$ expert is presented as a non-negative matrix of $n\times n$ rank, and each element of the $k$ matrix in equation $X^e=[x^k_{ij}]_{n\times n}$ denotes a non-negative number $x^e_{ij}$, where $1 \le k \le m$.

$$X^{e}=\begin{bmatrix} 0 & x_{12}^{e};x_{12}^{e'} & \cdots & x_{1n}^{e};x_{1n}^{e'} \\ x_{21}^{e};x_{21}^{e'} & 0 & \cdots & x_{2n}^{e};x_{2n}^{e'} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1}^{e};x_{n1}^{e'} & x_{n2}^{e};x_{n2}^{e'} & \cdots & 0 \end{bmatrix}_{nxn};\ 1\le i,j\le n;\ 1\le e\le m \tag{23}$$

where $x_{ij}^{e}$ and $x_{ij}^{e'}$ represent linguistic variables taken from the preliminary defined linguistic scale used by expert $e$ for the purpose of pairwise comparison.

In accordance with this, $X^1, X^2, \ldots, X^m$ matrices are judgment matrices of each of $m$ experts. The diagonal elements of the judgment matrix are all set to zero since the same factors do not influence each other.

If expert $k$ has a dilemma in the pairwise comparison of $(i,j)$, i.e. the expert $e$ cannot decide between two values from the linguistic scale, then both values are converted to matrix $X^e$. Then in position $(i,j)$ in matrix $X^e$ we have different $x^e_{ij}$ values, i.e. $x_{ij}^{e}\ne x_{ij}^{e'}$. If there is no uncertainty, then expert $k$ unambiguously selects one value. If such is the case, then the value of the position $(i,j)$, i.e. $x_{ij}^{e}=x_{ij}^{e'}$ is converted to a comparison matrix ($X^e$). For example, when comparing criteria in position (1,2), the expert cannot decide between two linguistic values (3 and 4, for example), then $x_{12}^{e}=3$, i.e. $x_{ij}^{e'}=4$ for position (1,2) in matrix $X^e$.

*Step 2. Calculate the average matrix*. Based on response matrices $X_k=[x^k_{ij}]_{n\times n}$ obtained from each $m$ expert, two matrices of aggregated sequence of experts $X^{*L}$ and $X^{*'U}$ are obtained.

$$X^{*L} = \begin{bmatrix} x_{11}^{1L}, x_{11}^{2L}, \ldots, x_{11}^{kL} & x_{12}^{1L}; x_{12}^{2L}; \ldots; x_{12}^{kL}, & \ldots, & x_{1n}^{1L}; x_{1n}^{2L}, \ldots, x_{1n}^{kL} \\ x_{21}^{1L}, x_{21}^{2L}, \ldots, x_{21}^{kL} & x_{22}^{1L}; x_{22}^{2L}; \ldots; x_{22}^{kL}, & \ldots, & x_{2n}^{1L}; x_{2n}^{2L}, \ldots, x_{2n}^{kL} \\ \ldots & \ldots & \ldots & \ldots \\ x_{n1}^{1L}, x_{n1}^{2L}, \ldots, x_{n1}^{kL} & x_{n2}^{1L}; x_{n2}^{2L}; \ldots; x_{n2}^{kL}, & \ldots, & x_{nn}^{1L}; x_{nn}^{2L}, \ldots, x_{nn}^{kL} \end{bmatrix} \tag{24}$$

$$X^{*'U} = \begin{bmatrix} x_{11}^{1'U}, x_{11}^{2'U}, \ldots, x_{11}^{k'U} & x_{12}^{1'U}; x_{12}^{2'U}; \ldots; x_{12}^{k'U}, & \ldots, & x_{1n}^{1'U}; x_{1n}^{2'U}, \ldots, x_{1n}^{k'U} \\ x_{21}^{1'U}, x_{21}^{2'U}, \ldots, x_{21}^{k'U} & x_{22}^{1'U}; x_{22}^{2'U}; \ldots; x_{22}^{k'U}, & \ldots, & x_{2n}^{1'U}; x_{2n}^{2'U}, \ldots, x_{2n}^{k'U} \\ \ldots & \ldots & \ldots & \ldots \\ x_{n1}^{1'U}, x_{n1}^{2'U}, \ldots, x_{n1}^{k'U} & x_{n2}^{1'U}; x_{n2}^{2'U}; \ldots; x_{n2}^{k'U}, & \ldots, & x_{nn}^{1'U}; x_{nn}^{2'U}, \ldots, x_{nn}^{k'U} \end{bmatrix} \tag{25}$$

where $x_{ij}^{L} = \left\{ x_{ij}^{1L}, x_{ij}^{2L}, \ldots, x_{ij}^{kL} \right\}$ and $x_{ij}^{'U} = \left\{ x_{ij}^{1'U}, x_{ij}^{2'U}, \ldots, x_{ij}^{k'U} \right\}$ denote the sequences used to describe the relative importance of criterion *i* in relation to criterion *j*. By applying equations (1) through (13), each sequence $x_{ij}^{k}$ and $x_{ij}^{k'}$ is converted to rough sequences $RN\left(x_{ij}^{kL}\right) = \left[ \underline{Lim}(x_{ij}^{kL}), \overline{Lim}(x_{ij}^{kL}) \right]$ and $RN\left(x_{ij}^{k'U}\right) = \left[ \underline{Lim}(x_{ij}^{k'U}), \overline{Lim}(x_{ij}^{k'U}) \right]$, where $\underline{Lim}(x_{ij}^{kL})$ and $\underline{Lim}(x_{ij}^{k'U})$ represent the lower limit, and $\overline{Lim}(x_{ij}^{kL})$ and $\overline{Lim}(x_{ij}^{k'U})$ upper limit of rough sequences $RN\left(x_{ij}^{kL}\right)$ and $RN\left(x_{ij}^{k'U}\right)$ respectively.

These rough sequences are defined in matrices (24) and (25). Thus we obtain $X^{1L}$, $X^{2L}$, …, $X^{mL}$ rough matrices (where *m* denotes the number of experts) for the first rough sequence $RN\left(x_{ij}^{kL}\right)$ and $X^{1'U}$, $X^{2'U}$, …, $X^{m'U}$ (where *m* denotes the number of experts) for the second rough sequence $RN\left(x_{ij}^{k'U}\right)$. Therefore for the first group of rough matrices $X^{1L}$, $X^{2L}$, …,$X^{mL}$ in position (*i*,*j*) we obtain rough sequence $RN\left(x_{ij}^{L}\right) = \left\{ \left[ \underline{Lim}(x_{ij}^{1L}), \overline{Lim}(x_{ij}^{1L}) \right], \left[ \underline{Lim}(x_{ij}^{2L}), \overline{Lim}(x_{ij}^{2L}) \right], ..., \left[ \underline{Lim}(x_{ij}^{mL}), \overline{Lim}(x_{ij}^{mL}) \right] \right\}$.

In the same way, for the second rough matrices $X^{1'U}$, $X^{2'U}$, …,$X^{m'U}$ in position (*i*,*j*) we obtain rough sequence $RN\left(x_{ij}^{'U}\right) = \left\{ \left[ \underline{Lim}(x_{ij}^{1'U}), \overline{Lim}(x_{ij}^{1'U}) \right], \left[ \underline{Lim}(x_{ij}^{2'U}), \overline{Lim}(x_{ij}^{2'U}) \right], ..., \left[ \underline{Lim}(x_{ij}^{m'U}), \overline{Lim}(x_{ij}^{m'U}) \right] \right\}$.

By applying equations (26) and (27) the mean rough sequences are as follows

$$RN(z_{ij}^{L}) = RN(x_{ij}^{1L}, x_{ij}^{2L}, ..., x_{ij}^{eL}) = \begin{cases} z_{ij}^{L} = \dfrac{1}{m} \sum_{e=1}^{m} x_{ij}^{eL} \\ z_{ij}^{U} = \dfrac{1}{m} \sum_{e=1}^{m} x_{ij}^{eU} \end{cases} \tag{26}$$

$$RN(z_{ij}^{'U}) = RN(x_{ij}^{1'U}, x_{ij}^{2'U}, ..., x_{ij}^{e'U}) = \begin{cases} z_{ij}^{'L} = \dfrac{1}{m} \sum_{e=1}^{m} x_{ij}^{e'L} \\ z_{ij}^{'U} = \dfrac{1}{m} \sum_{e=1}^{m} x_{ij}^{e'U} \end{cases} \tag{27}$$

Where *e* denotes the *e*-th expert ($e = 1, 2, ..., m$), $RN(z_{ij}^{L})$ and $RN(z_{ij}^{'U})$ represent rough sequences that at the same time respectively represent the lower and upper limit of the interval rough number $IRN(z_{ij})$, i.e. $IRN(z_{ij}) = \left[ RN(z_{ij}^{L}), RN(z_{ij}^{'U}) \right]$.

Thus, the average interval rough matrix of average responses *Z* is obtained

$$Z = \begin{bmatrix} 0 & IRN(z_{12}) & \cdots & IRN(z_{1n}) \\ IRN(z_{21}) & 0 & \cdots & IRN(z_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ IRN(z_{n1}) & IRN(z_{n2}) & \cdots & 0 \end{bmatrix} \tag{28}$$

Matrix *Z* denotes the starting effects caused by a specific factor as well as the starting effects obtained from other factors. The sum of each *i*-th row of matrix *Z* is the total direct effect that *I* delivers to other factors and the sum of each *j*-th column of matrix *Z* is the total direct effect that factor *j* receives from other factors.

*Step 3*. Based on matrix $Z$, a normalized initial direct-relation matrix $D = \left[ IRN(d_{ij}) \right]_{n \times n}$ is obtained, equation (29). By normalization, each element in matrix *D* is assigned a value between zero and one. The *D* matrix is obtained when each element $IRN(z_{ij})$ of matrix $Z$ is divided by rough number $IRN(s)$, as shown in equations (29) through (32)

$$D=\begin{bmatrix} 0 & IRN(d_{12}) & \cdots & IRN(d_{1n}) \\ IRN(d_{21}) & 0 & \cdots & IRN(d_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ IRN(d_{n1}) & IRN(d_{n2}) & \cdots & 0 \end{bmatrix} \tag{29}$$

where $IRN(d_{ij})$ is obtained by applying equation (30)

$$IRN(d_{ij})=\frac{IRN(z_{ij})}{IRN(s)}=IRN\left(\left[\frac{z_{ij}^{L}}{s_{ij}^{L}},\frac{z_{ij}^{U}}{s_{ij}^{U}}\right],\left[\frac{z_{ij}^{'L}}{s_{ij}^{'L}},\frac{z_{ij}^{'U}}{s_{ij}^{'U}}\right]\right) \tag{30}$$

The value of interval rough number $IRN(s)$ is obtained by applying equations (31) and (32)

$$\begin{aligned} IRN(s)&=\max\left(\sum\nolimits_{j=1}^{n} IRN(z_{ij})\right)=\left(\max\left[\sum\nolimits_{j=1}^{n}\underline{Lim}(z_{ij}),\sum\nolimits_{j=1}^{n}\overline{Lim}(z_{ij})\right],\max\left[\sum\nolimits_{j=1}^{n}\underline{Lim}(z_{ij}^{'}),\sum\nolimits_{j=1}^{n}\overline{Lim}(z_{ij}^{'})\right]\right) \\ &=\left(\left[\max\left(\sum\nolimits_{j=1}^{n}\underline{Lim}(z_{ij})\right),\max\left(\sum\nolimits_{j=1}^{n}\overline{Lim}(z_{ij})\right)\right],\left[\max\left(\sum\nolimits_{j=1}^{n}\underline{Lim}(z_{ij}^{'})\right),\max\left(\sum\nolimits_{j=1}^{n}\overline{Lim}(z_{ij}^{'})\right)\right]\right) \end{aligned} \tag{31}$$

i.e.

$$IRN(s)=\left(\left[\max\left\{\sum\nolimits_{j=1}^{n} z_{ij}^{L}\right\},\max\left\{\sum\nolimits_{j=1}^{n} z_{ij}^{U}\right\}\right],\left[\max\left\{\sum\nolimits_{j=1}^{n} z_{ij}^{'L}\right\},\max\left\{\sum\nolimits_{j=1}^{n} z_{ij}^{'U}\right\}\right]\right) \tag{32}$$

*Step 4.* By applying equations (33) through (35), the total-relation matrix $T=\left[IRN(t_{ij})\right]_{n\times n}$) of rank $n\times n$ is calculated, where $I$ denotes the identity matrix of the *nxn* rank. The element $IRN(t_{ij})$ denotes a direct influence of factor *i* on factor *j*, while *T* matrix denotes total relations among each pair of factors.

Since each interval rough number is composed of two rough sequences, and every rough sequence includes an upper and lower approximation, then the normalized matrix of average perception $D=\left[IRN(d_{ij})\right]_{n\times n}$ can be divided into four sub-matrices, i.e. $D=\left(\left[D^{L},D^{U}\right],\left[D^{'L},D^{'U}\right]\right)$, where $D^{L}=\left[\underline{Lim}(d_{ij})\right]_{n\times n}$, $D^{U}=\left[\overline{Lim}(d_{ij})\right]_{n\times n}$, $D^{'L}=\left[\underline{Lim}(d_{ij}^{'})\right]_{n\times n}$ and $D^{'U}=\left[\overline{Lim}(d_{ij}^{'})\right]_{n\times n}$. Moreover, $\lim_{m\to\infty}\left(D^{L}\right)^{m}=O$, $\lim_{m\to\infty}\left(D^{U}\right)^{m}=O$, $\lim_{m\to\infty}\left(D^{'L}\right)^{m}=O$ and $\lim_{m\to\infty}\left(D^{'U}\right)^{m}=O$, where $O$ denotes a zero matrix.

$$\left.\begin{aligned} &\lim_{m\to\infty}\left(I+D^{L}+D^{2L}+\ldots+D^{mL}\right)=\left(I-D^{L}\right)^{-1} \\ &\lim_{m\to\infty}\left(I+D^{U}+D^{2U}+\ldots+D^{mU}\right)=\left(I-D^{U}\right)^{-1} \\ &\lim_{m\to\infty}\left(I+D^{'L}+D^{2'L}+\ldots+D^{m'L}\right)=\left(I-D^{'L}\right)^{-1} \\ &\textit{and} \\ &\lim_{m\to\infty}\left(I+D^{'U}+D^{2'U}+\ldots+D^{m'U}\right)=\left(I-D^{'U}\right)^{-1} \end{aligned}\right\} \tag{33}$$

Therefore, the matrix of the total influences *T* will be obtained by calculating of the following elements

$$\left.\begin{aligned} &T^{L}=\lim_{m\to\infty}\left(I+D^{L}+D^{2L}+\ldots+D^{mL}\right)=\left(I-D^{L}\right)^{-1}=\left[\underline{Lim}(t_{ij}^{L})\right]_{n\times n} \\ &T^{U}=\lim_{m\to\infty}\left(I+D^{U}+D^{2U}+\ldots+D^{mU}\right)=\left(I-D^{U}\right)^{-1}=\left[\underline{Lim}(t_{ij}^{U})\right]_{n\times n} \\ &T^{'L}=\lim_{m\to\infty}\left(I+D^{'L}+D^{2'L}+\ldots+D^{m'L}\right)=\left(I-D^{'L}\right)^{-1}=\left[\underline{Lim}(t_{ij}^{'L})\right]_{n\times n} \\ &\textit{and} \\ &T^{'U}=\lim_{m\to\infty}\left(I+D^{'U}+D^{2'U}+\ldots+D^{m'U}\right)=\left(I-D^{'U}\right)^{-1}=\left[\underline{Lim}(t_{ij}^{'U})\right]_{n\times n} \end{aligned}\right\} \tag{34}$$

where $D^{L}=\left[\underline{Lim}(d_{ij})\right]_{n\times n}$, $D^{U}=\left[\overline{Lim}(d_{ij})\right]_{n\times n}$, $D^{'L}=\left[\underline{Lim}(d_{ij}^{'})\right]_{n\times n}$ and $D^{'U}=\left[\overline{Lim}(d_{ij}^{'})\right]_{n\times n}$.

Sub-matrices $T^L$, $T^U$, $T^{'L}$ and $T^{'U}$ together represent the interval rough matrix of the total influences $T=\left(\left[T^L,T^U\right],\left[T^{'L},T^{'U}\right]\right)$. Based on equations (33) and (34), a total-relation matrix is defined:

$$T=\begin{bmatrix} IRN(t_{11}) & IRN(t_{12}) & \cdots & IRN(t_{1n}) \\ IRN(t_{21}) & IRN(t_{22}) & \cdots & IRN(t_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ IRN(t_{n1}) & IRN(t_{n2}) & \cdots & IRN(t_{nn}) \end{bmatrix} \tag{35}$$

where $IRN(t_{ij})=\left[RN(t_{ij}^L),RN(t_{ij}^U)\right]$ is an interval rough number used to express the indirect effects of factor $i$ on factor $j$. Then matrix $T$ reflects the inter-dependence of each pair of factors.

*Step 5.* Calculating the sum of rows and columns of total-relation matrix *T*. In total-relation matrix *T*, the sum of rows and sum of columns are denoted as vectors *R* and *C*, rank *n×1*:

$$IRN(R_i)=\left[\sum_{j=1}^{n} IRN(t_{ij})\right]_{n\times 1}=\left[\left(\left[\sum\nolimits_{j=1}^{n} t_{ij}^L,\sum\nolimits_{j=1}^{n} t_{ij}^U\right],\left[\sum\nolimits_{j=1}^{n} t_{ij}^{'L},\sum\nolimits_{j=1}^{n} t_{ij}^{'U}\right]\right)\right]_{n\times 1} \tag{36}$$

$$IRN(C_i)=\left[\sum_{i=1}^{n} IRN(t_{ij})\right]_{1\times n}=\left[\left(\left[\sum\nolimits_{i=1}^{n} t_{ij}^L,\sum\nolimits_{i=1}^{n} t_{ij}^U\right],\left[\sum\nolimits_{i=1}^{n} t_{ij}^{'L},\sum\nolimits_{i=1}^{n} t_{ij}^{'U}\right]\right)\right]_{1\times n} \tag{37}$$

The value $R_i$ denotes the sum of the *i*-th row of matrix *T* and shows the total direct and indirect effects that criterion *I* delivers to other factors. Similarly, the value $C_i$ is the sum of the *j*-th column of matrix *T*, and represents the total direct and indirect effects that factor *j* receives from other factors. In cases where $i=j$, equation ($R_i+C_i$) indicates the impact of the factors and equation ($R_i$-$C_i$) indicates the intensity of the factors compared to others (Pamučar and Ćirović, 2015).

*Step 6.* Setting a threshold value ($\alpha$) and constructing a cause-and-effect relationship diagram. The threshold value ($\alpha$) is determined by applying equation (38) which gives the mean of the elements in matrix *T*.

$$\alpha=\frac{\sum\nolimits_{i=1}^{n}\sum\nolimits_{j=1}^{n}\left[IRN(t_{ij})\right]}{N} \tag{38}$$

where *N* denotes the number of matrix elements (35).

The construction of a cause-and-effect diagram visualizes the complex interrelationship and provides information in order to determine the most important factors and how they influence the affected factors. Factors $t_{ij}$ with a value higher than threshold value $\alpha$ are selected and shown in the cause-and-effect diagram.

Elements of matrix *T* with values higher than the threshold $\alpha$ are selected and shown in the diagram where the *x*-axis represents $IRN(R_i+C_i)$, and the *y*-axis $IRN(R_i$-$C_i)$ and they are used to denote the relationship between two factors. When presenting the factor relationships, the arrow of the cause-and-effect relationship will be directed from the factor with a value lower than threshold value $\alpha$ to the element with a value higher than threshold value $\alpha$.

Weight coefficients of the clusters/criteria are calculated once the cause-and-effect relationship diagram (CERD) is constructed by applying the Analytic Network Process (ANP).

### *4.2. The IR-ANP method*

ANP is a generalized AHP method that, unlike hierarchy structured models, takes into account different forms of dependency and feedback. The structure of the feedback is not linear and is closer to a network in which interdependent loops frequently appear. Matrices that describe these dependences are called supermatrices and should always follow the column stochastic principle, meaning that the sum of elements in each column should be equal to one (Saaty and Vargas, 2012).

Calculating the relative weights of criteria using traditional ANP means that the levels of interdependence of the factors are treated as reciprocal values. In contrast, in using the DEMATEL method, the levels of interdependence of factors do not have reciprocal values, which is closer to real circumstances (Yang & Tzeng, 2011). The following section deals with a novel approach, which integrates the IR-DEMATEL method into the IR-ANP method (IRD'ANP model). This integration is carried out as follows:

*Step 1*. Developing an unweighted supermatrix. Prior to developing the unweighted supermatrix, a network model for the ANP method should be defined based on the total relation matrix and ERD.

An unweighted supermatrix is created when each level with the total degree of influence from the total relation matrix $T$ is normalized by IR'DEMATEL. To normalize the matrix, it is necessary to determine the sum of elements of the matrix by columns.

$$T_c = \begin{array}{cc} & \begin{array}{cccc} D_1 & D_2 & & D_n \\ c_{11}c_{12}\cdots c_{1m1} & c_{21}c_{22}\cdots c_{2m2} & \cdots & c_{n1}c_{n2}\cdots c_{nmn} \end{array} \\ \begin{array}{cc} D_1 & \begin{array}{c} c_{11} \\ c_{12} \\ \vdots \\ c_{1m1} \end{array} \\ D_2 & \begin{array}{c} c_{21} \\ c_{22} \\ \vdots \\ c_{2m2} \end{array} \\ D_n & \begin{array}{c} c_{n1} \\ c_{n2} \\ \vdots \\ c_{nmn} \end{array} \end{array} & \begin{bmatrix} T_c^{11} & T_c^{12} & \cdots & T_c^{1n} \\ T_c^{21} & T_c^{22} & \cdots & T_c^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ T_c^{n1} & T_c^{n2} & \cdots & T_c^{nn} \end{bmatrix} \end{array} \qquad (39)$$

where matrix $T_c^{11}$ contains factors from group D1 and influences factors from group D1. Matrix $T_c^{21}$ (40) contains factors from the group (criteria) D2 and influences with respect to the factors from group D2, etc.

$$T_c^{12} = \begin{bmatrix} IRN(t_{c^{12}}^{12}) & \ldots & IRN(t_{c^{1j}}^{12}) & \ldots & IRN(t_{c^{1m1}}^{12}) \\ \vdots & & \vdots & \ldots & \vdots \\ IRN(t_{c^{i1}}^{12}) & \ldots & IRN(t_{c^{ij}}^{12}) & \ldots & IRN(t_{c^{1m1}}^{12}) \\ \vdots & & \vdots & & \vdots \\ IRN(t_{c^{m12}}^{12}) & \ldots & IRN(t_{c^{m1j}}^{12}) & \ldots & IRN(t_{c^{m1m1}}^{12}) \end{bmatrix} \qquad (40)$$

*Step 2*. The normalized total influence matrix for criteria $T_c^{\alpha}$. Normalization takes place once $T_c$ is developed. During the normalization process, the total-influence matrix $T_c$ yields $T_c^{\alpha}$. The normalized matrix $T_c^{\alpha}$ is shown below (41)

$$T_c^{\alpha} = \begin{array}{cc} & \begin{array}{cccc} D_1 & D_2 & & D_n \\ c_{11}c_{12}\cdots c_{1m1} & c_{21}c_{22}\cdots c_{2m2} & \cdots & c_{n1}c_{n2}\cdots c_{nmn} \end{array} \\ \begin{array}{cc} D_1 & \begin{array}{c} c_{11} \\ c_{12} \\ \vdots \\ c_{1m1} \end{array} \\ D_2 & \begin{array}{c} c_{21} \\ c_{22} \\ \vdots \\ c_{2m2} \end{array} \\ D_n & \begin{array}{c} c_{n1} \\ c_{n2} \\ \vdots \\ c_{nmn} \end{array} \end{array} & \begin{bmatrix} T_c^{\alpha 11} & T_c^{\alpha 212} & \cdots & T_c^{\alpha 1n} \\ T_c^{\alpha 21} & T_c^{\alpha 22} & \cdots & T_c^{\alpha 2n} \\ \vdots & \vdots & \ddots & \vdots \\ T_c^{\alpha n1} & T_c^{\alpha n2} & \cdots & T_c^{\alpha nn} \end{bmatrix} \end{array} \qquad (41)$$

To explain this is the normalization of $T_c^{\alpha 11}$ on dimension D1. The sum of factors $c_{11}, \ldots, c_{1m1}$ within group D1 is obtained by applying the following equation:

$$IRN(d_{ci}^{11}) = \sum_{j=1}^{m_1} IRN(t_{ij}^{11}),\ i = 1, 2, \ldots, m_1 \qquad (42)$$

where $IRN(t_{cj}^{11})$ denotes the values of factor influences $c_{11}, \ldots, c_{1m1}$ in relation to factors from group D1, and $IRN(t_{c11}^{\alpha 11})$ elements denote their normalized values.

*Step 3*. Developing unweighted supermatrix *W*. Since the total influence matrix $T_c$ fills the interdependence between the dimensions and criteria, we can transpose the normalized total influence matrix $T_c^{\alpha}$ by the dimensions based on the basic concept of ANP resulting in unweighted supermatrix $W = [T_c^{\alpha}]'$, equation (43)

$$W = (T_c^\alpha)' = \begin{array}{ccc} & & \begin{array}{cccc} D_1 & D_2 & & D_n \\ c_{11}c_{12}\dots c_{1m1} & c_{21}c_{22}\dots c_{2m2} & \dots & c_{n1}c_{n2}\dots c_{nmn} \end{array} \\ D_1 & \begin{array}{c} c_{11} \\ c_{12} \\ \vdots \\ c_{1m1} \end{array} & \\ D_2 & \begin{array}{c} c_{21} \\ c_{22} \\ \vdots \\ c_{2m2} \end{array} & \begin{bmatrix} W^{11} & W^{12} & \cdots & W^{1n} \\ W^{21} & W^{22} & \cdots & W^{2n} \\ \vdots & \vdots & \ddots & \vdots \\ W^{n1} & W^{n1} & \cdots & W^{nn} \end{bmatrix} \\ D_n & \begin{array}{c} c_{n1} \\ c_{n2} \\ \vdots \\ c_{nmn} \end{array} & \end{array} \tag{43}$$

where $W^{11}$ matrix denotes values of factor influences from D1 group in relation to factors from group D1.

*Step 4*. Developing weighted normalized supermatrix $W^\alpha$. Elements of weighted normalized supermatrix $W^\alpha$ are obtained by multiplying elements of unweighted supermatrix $W$ and appropriate elements of the normalized total influence matrix $T_D^\alpha$. Elements of the normalized total influence matrix $T_D^\alpha$ are obtained by normalizing the total influence matrix $T_D$, as stated below (44).

$$T_D^\alpha = \begin{bmatrix} IRN(t_D^{\alpha 11}) & \dots & IRN(t_D^{\alpha 1j}) & \dots & IRN(t_D^{\alpha 1n}) \\ \vdots & & \vdots & \dots & \vdots \\ IRN(t_D^{\alpha i1}) & \dots & IRN(t_D^{\alpha ij}) & \dots & IRN(t_D^{\alpha in}) \\ \vdots & & \vdots & & \vdots \\ IRN(t_D^{\alpha n1}) & \dots & IRN(t_D^{\alpha nj}) & \dots & IRN(t_D^{\alpha nn}) \end{bmatrix} \tag{44}$$

where $IRN(t_D^{\alpha j}) = IRN(t_D^j)/IRN(d_i)$, and value of $IRN(d_i)$ will be obtained as $IRN(d_i) = \sum_{j=1}^{n} IRN(t_D^{ij})$.

Once the elements of matrix $T_D^\alpha$ are obtained, the elements of new weighted supermatrix $W^\alpha$ are calculated. The elements of matrix $W^\alpha$ are obtained by multiplying the normalized total influence matrix of the dimensions $T_D^\alpha$ and unweighted supermatrix $W$.

*Step 5*. Finding the limit of weighted supermatrix $W^\alpha$. The weighted supermatrix is multiplied by itself multiple times to obtain a limit supermatrix, then the weight of each criteria is obtained. The weighted supermatrix can be raised to the limiting powers until the supermatrix has converged and become a long-term stable supermatrix to obtain global priority vectors, called IRD'ANP influence weights, such as $\lim_{k\to\infty} = W^k$, where $W$ denotes the limit supermatix, while $k$ represents any number.

### *4.3. The IR-MAIRCA method*

The basic assumption of the MAIRCA method is to determine the gap between the ideal and empirical weights. Summing the gaps for each criterion gives the total gap for every alternative observed. Finally, the alternatives are ranked, and the best ranked alternative is the one with the smallest value of the total gap. The MAIRCA method has 7 steps (Pamučar et al., 2014; Gigović et al., 2016):

*Step 1*. Forming the initial decision matrix ($Y$). The first step includes evaluation of $l$ alternatives per $n$ criteria. Based on response matrices $Y_k=[y^k_{ij}]_{l\times n}$ by all $m$ experts we obtain two matrices of the aggregated sequences of $Y^{*L}$ and $Y^{*'U}$ experts

$$Y^{*L} = \begin{bmatrix} y_{11}^{1L}, y_{11}^{2L}, \dots, y_{11}^{kL} & y_{12}^{1L}; y_{12}^{2L}; \dots; y_{12}^{kL}, & \dots, & y_{1n}^{1L}; y_{1n}^{2L}, \dots, y_{1n}^{kL} \\ y_{21}^{1L}, y_{21}^{2L}, \dots, y_{21}^{kL} & y_{22}^{1L}; y_{22}^{2L}; \dots; y_{22}^{kL}, & \dots, & y_{2n}^{1L}; y_{2n}^{2L}, \dots, y_{2n}^{kL} \\ \dots & \dots & \dots & \dots \\ y_{n1}^{1L}, y_{n1}^{2L}, \dots, y_{n1}^{kL} & y_{n2}^{1L}; y_{n2}^{2L}; \dots; y_{n2}^{kL}, & \dots, & y_{nn}^{1L}; y_{nn}^{2L}, \dots, y_{nn}^{kL} \end{bmatrix} \tag{45}$$

$$Y^{*'U} = \begin{bmatrix} y_{11}^{1'U}, y_{11}^{2'U}, \dots, y_{11}^{k'U} & y_{12}^{1'U}; y_{12}^{2'U}; \dots; y_{12}^{k'U}, & \dots, & y_{1n}^{1'U}; y_{1n}^{2'U}, \dots, y_{1n}^{k'U} \\ y_{21}^{1'U}, y_{21}^{2'U}, \dots, y_{21}^{k'U} & y_{22}^{1'U}; y_{22}^{2'U}; \dots; y_{22}^{k'U}, & \dots, & y_{2n}^{1'U}; y_{2n}^{2'U}, \dots, y_{2n}^{k'U} \\ \dots & \dots & \dots & \dots \\ y_{n1}^{1'U}, y_{n1}^{2'U}, \dots, y_{n1}^{k'U} & y_{n2}^{1'U}; y_{n2}^{2'U}; \dots; y_{n2}^{k'U}, & \dots, & y_{nn}^{1'U}; y_{nn}^{2'U}, \dots, y_{nn}^{k'U} \end{bmatrix} \tag{46}$$

where $y_{ij}^{L} = \left\{ y_{ij}^{1L}, y_{ij}^{2L}, \ldots, y_{ij}^{kL} \right\}$ and $y_{ij}^{'U} = \left\{ y_{ij}^{1'U}, y_{ij}^{2'U}, \ldots, y_{ij}^{k'U} \right\}$ denote the sequences for describing the relative importance of criterion *i* in relation to alternative *j*. By applying equations (1) through (13), sequences $y_{ij}^{k}$ and $y_{ij}^{k'}$ are transformed into rough sequences $RN\left(y_{ij}^{kL}\right)$ and $RN\left(y_{ij}^{k'U}\right)$. Consequently, rough matrices $Y^{1L}$, $Y^{2L}$, …,$Y^{mL}$ are obtained for the first rough sequence $RN\left(y_{ij}^{kL}\right)$ and $Y^{1'U}$, $Y^{2'U}$, …, $Y^{m'U}$ for other rough sequence $RN\left(y_{ij}^{k'U}\right)$, where *m* denotes the number of experts. Therefore, for the first group of rough matrices $Y^{1L}$, $Y^{2L}$, …,$Y^{mL}$ we obtain rough sequences
$RN\left(y_{ij}^{L}\right) = \left\{ \left[ \underline{Lim}(y_{ij}^{1L}), \overline{Lim}(y_{ij}^{1L}) \right], \left[ \underline{Lim}(y_{ij}^{2L}), \overline{Lim}(y_{ij}^{2L}) \right], ..., \left[ \underline{Lim}(y_{ij}^{mL}), \overline{Lim}(y_{ij}^{mL}) \right] \right\}$, i.e. for the second group of rough matrices $Y^{1L}$, $Y^{2L}$, …,$Y^{mL}$ we obtain rough sequence
$RN\left(y_{ij}^{'U}\right) = \left\{ \left[ \underline{Lim}(y_{ij}^{1'U}), \overline{Lim}(y_{ij}^{1'U}) \right], \left[ \underline{Lim}(y_{ij}^{2'U}), \overline{Lim}(y_{ij}^{2'U}) \right], ..., \left[ \underline{Lim}(y_{ij}^{m'U}), \overline{Lim}(y_{ij}^{m'U}) \right] \right\}$.
By applying equations (47) and (48) we obtain mean rough sequences

$$RN(y_{ij}^{L}) = RN(y_{ij}^{1L}, y_{ij}^{2L}, ..., y_{ij}^{eL}) = \begin{cases} y_{ij}^{L} = \dfrac{1}{m}\sum_{e=1}^{m} y_{ij}^{eL} \\ y_{ij}^{U} = \dfrac{1}{m}\sum_{e=1}^{m} y_{ij}^{eU} \end{cases} \tag{47}$$

$$RN(y_{ij}^{'U}) = RN(y_{ij}^{1'U}, y_{ij}^{2'U}, ..., y_{ij}^{e'U}) = \begin{cases} y_{ij}^{'L} = \dfrac{1}{m}\sum_{e=1}^{m} y_{ij}^{e'L} \\ y_{ij}^{'U} = \dfrac{1}{m}\sum_{e=1}^{m} y_{ij}^{e'U} \end{cases} \tag{48}$$

Where *e* denotes the *e*-th expert ($e = 1, 2, ..., m$), $RN(z_{ij}^{L})$ and $RN(z_{ij}^{'U})$ denote the rough sequences of interval rough number $IRN(z_{ij}) = \left[ RN(z_{ij}^{L}), RN(z_{ij}^{'U}) \right]$.

In such a way, interval rough vectors $A_i = \left( IRN\left(y_{i1}\right), IRN\left(y_{i2}\right), ..., IRN\left(y_{in}\right) \right)$ of the mean initial decision matrix are obtained, where $IRN(y_{ij}) = \left[ RN(y_{ij}^{L}), RN(y_{ij}^{'U}) \right] = \left( \left[ y_{ij}^{L}, y_{ij}^{U} \right], \left[ y_{ij}^{'L}, y_{ij}^{'U} \right] \right)$ denotes the value of the $i$ -th alternative as per the $j$ -th criterion ($i = 1, 2, ..., l$; $j = 1, 2, ..., n$).

$$Y = \begin{array}{c} \\ A_1 \\ A_2 \\ ... \\ A_l \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & ... & C_n \end{array} \\ \begin{bmatrix} IRN(y_{11}) & IRN(y_{12}) & ... & IRN(y_{1n}) \\ IRN(y_{21}) & IRN(y_{22}) & & IRN(y_{2n}) \\ ... & ... & ... & ... \\ IRN(y_{l1}) & IRN(y_{l2}) & ... & IRN(y_{ln}) \end{bmatrix}_{l \times n} \end{array} \tag{49}$$

where *l* denotes the number of alternatives, and *n* denotes the total sum of the criteria.
*Step 2.* Defining the preferences according to the selection of alternatives $P_{A_i}$. When selecting an alternative, a decision maker (DM) is neutral, i.e. does not have preferences for any of the proposed alternatives. Since any alternative can be chosen with equal probability, the preference per selection of one of *l* possible alternatives is as follows

$$P_{A_i} = \frac{1}{l}; \sum_{i=1}^{l} P_{A_i} = 1, \ i = 1, 2, ..., l \tag{50}$$

where *l* denotes the number of alternatives.
*Step 3.* Calculating the theoretical evaluation matrix elements ($T_p$). Theoretical evaluation matrix ($T_p$) is developed in $l \ x \ n$ format (*l* denotes the number of alternatives, *n* denotes the number of criteria). Theoretical evaluation matrix elements ($IRN(t_{pij})$) are calculated as the multiplication of the preferences according to alternatives $P_{A_i}$ and criteria weights ($IRN(w_i)$, $i = 1, 2, ..., n$) obtained by applying the IR'DEMATEL method.

$$T_p = \begin{array}{c} \\ P_{A_1} \\ P_{A_2} \\ \ldots \\ P_{A_l} \end{array} \begin{array}{c} \begin{array}{cccc} IRN(w_1) & IRN(w_2) & \ldots & IRN(w_n) \end{array} \\ \begin{bmatrix} IRN(t_{p11}) & IRN(t_{p12}) & \ldots & IRN(t_{p1n}) \\ IRN(t_{p21}) & IRN(t_{p22}) & & IRN(t_{p2n}) \\ \ldots & \ldots & \ldots & \ldots \\ IRN(t_{pl1}) & IRN(t_{pl2}) & \ldots & IRN(t_{pln}) \end{bmatrix}_{l \times n} \end{array} \tag{51}$$

where $P_{A_i}$ denotes the preferences per selection of alternatives, $IRN(w_i)$ the weight coefficients of the evaluation criteria, and $IRN(t_{pij})$ the theoretical assessment of the alternatives for the evaluation criterion. Elements constituting the matrix $T_p$ are then defined by applying equation (52)

$$t_{pij} = P_{Ai} \cdot IRN(w_i) = P_{Ai} \cdot \left[ RN(w_i^L), RN(w_i^U) \right] \tag{52}$$

Since the DM is neutral to the initial selection of alternatives, all preferences ( $P_{A_i}$ ) are equal for all alternatives. Since preferences ( $P_{A_i}$ ) are equal for all alternatives, then matrix (51) will have $1\,x\,n$ format ($n$ denotes the number of criteria).

$$T_p = P_{A_i} \begin{array}{c} \begin{array}{ccc} IRN(w_1) & IRN(w_2) & \ldots \quad IRN(w_n) \end{array} \\ \left[ \left( \left[ t_{p1}^L, t_{p1}^U \right], \left[ t_{p1}^{'L}, t_{p1}^{'U} \right] \right) \;\; \left( \left[ t_{p2}^L, t_{p2}^U \right], \left[ t_{p2}^{'L}, t_{p2}^{'U} \right] \right) \;\; \ldots \;\; \left( \left[ t_{pn}^L, t_{pn}^U \right], \left[ t_{pn}^{'L}, t_{pn}^{'U} \right] \right) \right]_{1xn} \end{array} \tag{53}$$

where $n$ denotes the number of criteria, $P_{A_i}$ the preferences according to the selection of alternatives, $w_i$ the weight coefficients of the evaluation criteria.

*Step 4.* Determining the real evaluation matrix ( $T_r$ ). Calculation of the real evaluation matrix elements ( $T_r$ ) is done by multiplying the real evaluation matrix elements ( $T_p$ ) and elements of the initial decision matrix ( $X$ ) according to the following equation:

$$IRN(t_{rij}) = IRN(t_{pij}) \cdot IRN(x_{nij}) = \left( \left[ t_{pij}^L, t_{pij}^U \right], \left[ t_{pij}^{'L}, t_{pij}^{'U} \right] \right) \cdot \left( \left[ \hat{y}_{ij}^L, \hat{y}_{ij}^U \right], \left[ \hat{y}_{ij}^{'L}, \hat{y}_{ij}^{'U} \right] \right) \tag{54}$$

where $IRN(t_{pij})$ denotes elements of the theoretical assessment matrix, and $IRN(\hat{y}_{ij})$ denotes elements of normalized matrix $Y = \left[ IRN(\hat{y}_{ij}) \right]_{l \times n}$. Normalization of the mean initial decision matrix (49) is carried out by applying equations (55) and (56)

a) For “benefit” type criteria (a higher criterion value is preferable)

$$IRN(\hat{y}_{ij}) = \left( \left[ \hat{y}_{ij}^L, \hat{y}_{ij}^U \right], \left[ \hat{y}_{ij}^{'L}, \hat{y}_{ij}^{'U} \right] \right) = \left( \left[ \frac{y_{ij}^L - y_{ij}^-}{y_{ij}^+ - y_{ij}^-}, \frac{y_{ij}^U - y_{ij}^-}{y_{ij}^+ - y_{ij}^-} \right], \left[ \frac{y_{ij}^{'L} - y_{ij}^-}{y_{ij}^+ - y_{ij}^-}, \frac{y_{ij}^{'U} - y_{ij}^-}{y_{ij}^+ - y_{ij}^-} \right] \right) \tag{55}$$

b) For “cost” type criteria (a lower criterion value is preferable)

$$IRN(\hat{y}_{ij}) = \left( \left[ \hat{y}_{ij}^L, \hat{y}_{ij}^U \right], \left[ \hat{y}_{ij}^{'L}, \hat{y}_{ij}^{'U} \right] \right) = \left( \left[ \frac{y_{ij}^{'U} - y_{ij}^+}{y_{ij}^- - y_{ij}^+}, \frac{y_{ij}^{'L} - y_{ij}^+}{y_{ij}^- - y_{ij}^+} \right], \left[ \frac{y_{ij}^U - y_{ij}^+}{y_{ij}^- - y_{ij}^+}, \frac{y_{ij}^L - y_{ij}^+}{y_{ij}^- - y_{ij}^+} \right] \right) \tag{56}$$

where $y_i^-$ and $y_i^+$ denote the minimum and maximum values of the marked criterion by its alternatives, respectively:

$$y_{ij}^- = \min_j \left\{ y_{ij}^L, y_{ij}^{'L} \right\} \tag{57}$$

$$y_{ij}^+ = \max_j \left\{ y_{ij}^U, y_{ij}^{'U} \right\} \tag{58}$$

*Step 5.* Calculating the total gap matrix ( $G$ ). Elements of matrix $G$ are obtained as the difference (gap) between the theoretical ( $t_{pij}$ ) and real evaluations ( $t_{rij}$ ), or by actually subtracting the elements of the theoretical evaluation matrix ( $T_p$ ) from the elements of the real evaluation matrix ( $T_r$ )

$$G = T_p - T_r = \begin{bmatrix} IRN(g_{11}) & IRN(g_{12}) & \ldots & IRN(g_{1n}) \\ IRN(g_{21}) & IRN(g_{22}) & \ldots & IRN(g_{2n}) \\ \ldots & \ldots & \ldots & \ldots \\ IRN(g_{l1}) & IRN(g_{l2}) & \ldots & IRN(g_{ln}) \end{bmatrix}_{l \times n} \tag{59}$$

where $n$ denotes the number of criteria, $l$ denotes the number of alternatives, and $g_{ij}$ represents the gap for alternative $i$ as per criterion $j$. Gap $g_{ij}$ takes values from the interval rough number according to equation (60)

$$IRN(g_{ij}) = IRN(t_{pij}) - IRN(t_{r_{ij}}) = \left(\left[t_{pij}^{L}, t_{pij}^{U}\right], \left[t_{pij}^{'L}, t_{pij}^{'U}\right]\right) - \left(\left[t_{rij}^{L}, t_{rij}^{U}\right], \left[t_{rij}^{'L}, t_{rij}^{'U}\right]\right) \tag{60}$$

It is preferable that the $IRN(g_{ij})$ value goes to zero ($IRN(g_{ij}) \to 0$) since the alternative with the smallest difference between the theoretical ($IRN(t_{pij})$) and real evaluation ($IRN(t_{rij})$) is chosen. If alternative $A_i$ for criterion $C_i$ has a theoretical evaluation value equal to the real evaluation value ($IRN(t_{pij}) = IRN(t_{rij})$) then the gap for alternative $A_i$ for criterion $C_i$ is zero, i.e. alternative $A_i$ per criterion $C_i$ is the best (ideal) alternative.
If alternative $A_i$ for criterion $C_i$ has a theoretical evaluation value $IRN(t_{pij})$ and the real weight value is zero, then the gap for alternative $A_i$ for criterion $C_i$ is $IRN(g_{ij}) \approx IRN(t_{pij})$. This means that alternative $A_i$ for criterion $C_i$ is the worst (anti-ideal) alternative.

*Step 6*. Calculating the final values of the criteria functions ($Q_i$) per alternatives. The values of the criteria functions are obtained by summing the gaps from matrix (59) for each alternative as per evaluation criteria, i.e. by summing matrix elements ($G$) per columns as shown in equation (61)

$$IRN(Q_i) = \sum_{j=1}^{n} IRN(g_{ij}),\ i = 1, 2, ..., m \tag{61}$$

where *n* denotes the number of criteria, *m* denotes the number of chosen alternatives.
The alternatives can be ranked by applying the rules governing the ranking of interval rough numbers described in Section 3 or by converting interval rough numbers into real numbers.
The conversion of interval rough number $IRN(Q_i) = \left(\left[Q_i^{L}, Q_i^{U}\right], \left[Q_i^{'L}, Q_i^{'U}\right]\right)$ into real number $Q_i$ is enabled by applying equations (62) and (63). The intervals between the upper and lower limits for both object classes, equations (9) and (10), are used for defining indicator $\mu_i$ ($0 \le \mu_i \le 1$) which is used for converting the interval rough number into a real number.

$$\mu_i = \frac{RB(Q_{ui})}{RB(Q_{ui}) + RB(Q_{li})};\ \ RB(Q_{ui}) = Q_i^{'U} - Q_i^{'L};\ \ RB(Q_{li}) = Q_i^{U} - Q_i^{L} \tag{62}$$

$$Q_i = \mu_i \cdot Q_i^{L} + (1 - \mu_i) \cdot Q_i^{'U} \tag{63}$$

*Step 7*. Defining the dominance index of the best-ranked alternative ($A_{D,1-j}$) and the final rank of alternatives. The dominance index of the best-ranked alternative defines its advantage in relation to the other alternatives. The dominance index is determined by applying equation (64).

$$A_{D,1-j} = \left|\frac{|Q_j| - |Q_1|}{|Q_n|}\right|,\ \ j = 2, 3, .., m \tag{64}$$

where $Q_1$ denotes the criterion function of the best-ranked alternative, $Q_n$ denotes the criterion function of the last ranked alternative, $Q_j$ denotes the criterion function of the alternative which is compared to the best-ranked alternative, and *m* denotes the number of alternatives.
Once the dominance index is determined, the dominance threshold $I_D$ is determined by applying equation (65)

$$I_D = \frac{m-1}{m^2} \tag{65}$$

where *m* denotes the number of alternatives.
Provided that the dominance index $A_{D,1-j}$ is greater or equal to dominance threshold $I_D$ ($A_{D,1-j} \ge I_D$), the obtained rank will be retained. However, if the dominance index $A_{D,1-j}$ is smaller than the dominance threshold $I_D$ ($A_{D,1-j} < I_D$), then it cannot be said with certainty that the first ranked alternatives have an advantage over the alternative being analyzed. The said restrictions can be shown by applying the following equation

$$R_{final,j} = \begin{cases} A_{D,1-j} \ge I_D & \Rightarrow\ \ R_{final,j} = R_{initial,j} \\ A_{D,1-j} < I_D & \Rightarrow\ \ R_{final,j} = R_{initial,1} \end{cases} \tag{66}$$

where $R_{initial,j}$ denotes the initial rank of the alternative that is compared with the best-ranked alternative, $R_{final,j}$ denotes final rank of the alternative which is compared to the best-ranked alternative, $I_D$ denotes the dominance threshold, and $A_{D,1-j}$ denotes the dominance index of the best-ranked alternative in relation to the alternative.

Provided that criterion $A_{D,1-j} < I_D$ is satisfied, then the rank of the alternative that is compared to the best-ranked alternative will be corrected and then treated as the best-ranked alternative and assigned the value "1*". In this way it is emphasized that the best-ranked alternative is characterized by a smaller advantage than the one specified in equation (65).

Assume, for example, that the best-ranked alternative is compared to the second-ranked alternative and that the criterion $A_{D,1-2} < I_D$ is satisfied. Then the second-ranked alternative will be assigned rank "1*". The comparison may proceed with the third-ranked alternative. If for the third-ranked alternative criterion $A_{D,1-3} < I_D$ is satisfied, then the third-ranked alternative will be assigned rank "1**" and so on, until reaching the last alternative.

Finally, correction of the initial ranks ($R_{initial}$) is carried out for all alternatives satisfying criterion $A_{D,1-j} < I_D$, while the ranks of alternatives satisfying the criterion $A_{D,1-j} \geq I_D$ remain unchanged. Therefore, the final rank of alternatives ($R_{final}$) which is presented simultaneously with the initial rank of alternatives ($R_{initial}$) is obtained.

## 5. Application of the IRD'ANP-MAIRCA model: Bidder evaluation in the public procurement procedure

Application of the hybrid IRN'ANP-MAIRCA model is shown using the example of ten bidders who submitted their tenders in a public procurement procedure launched by the public administration of the Republic of Serbia (Service for centralized public procurements and control of procurements for the City of Belgrade). Based on the above-mentioned analyses, and for the purpose of bidder evaluation, 12 criteria were defined: Duration of procurement ($C1_1$), Degree of realization ($C1_2$), Price ($C1_3$), Quality of Packaging ($C2_4$), Quality Certificate ($C2_5$), Time of delivery ($C3_6$), Quantities Needed ($C3_7$), Warranty Period ($C4_8$), Service ($C4_9$), Available capacities and resources ($C5_{10}$), Human resources ($C5_{11}$), Technical experience of the staff ($C5_{12}$). The criteria were grouped into five clusters: Safety in realization (D1), Time of delivery (D3), Post-warranty period (D4), and Functional characteristics (D5). In criterion ($Ci_j$) *i* denotes the cluster with criterion *j* grouped within it.

In accordance with this (Figure 3), the hybrid IRN'ANP-MAIRCA model is demonstrated through three phases. Phase 1 applies the IR-DEMATEL model in order to determine relations among the evaluation criteria. Phase 2 deals with the results of the IR-DEMATEL model (CER diagram and Total Relation Matrix) that is used for calculating the interval rough weight coefficients of the criteria by applying the IR-ANP model. Finally, Phase 3 includes bidder evaluation by applying the IR-MAIRCA model. In addition to ranking, the evaluation of bidders also includes a comparison of the ranks obtained by other multi-criteria decision models (TOPSIS, VIKOR, MABAC, TODIM and ELECTRE I methods). The most appropriate bidder is selected based on the results obtained, Spearman's rank correlation coefficient, and a sensitivity analysis of the IRN'ANP-MAIRCA model. The procedure is shown below.

*Phase 1: IR-DEMATEL model*

The IR-DEMATEL model is used here for the expert analysis of the criteria. This research included eight experts each with a minimum of ten years of experience in public procurements. The following scale was used to evaluate the clusters/criteria; 1 – very low influence; 2 – low influence; 3 – moderate influence; 4 – high influence; 5 – very high influence. All of the experts participated in evaluating the clusters and criteria. Once the evaluation was completed, eight matrices were obtained for pairwise comparison of the criteria *12x12* in size (Table 3) and eight matrices for pairwise comparison of the clusters, *5x5* in size (Supplementary file Table 1S).

Table 3. Comparison of the evaluation criteria by experts

| | DM1 | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $C1_1$ | $C1_2$ | $C1_3$ | $C2_4$ | $C2_5$ | $C3_6$ | $C3_7$ | $C4_8$ | $C4_9$ | $C5_{10}$ | $C5_{11}$ | $C5_{12}$ |
| $C1_1$ | (0;0) | (3;4) | (3;3) | (2;3) | (2;3) | (3;5) | (3;3) | (2;3) | (2;5) | (1;1) | (4;4) | (4;4) |
| $C1_2$ | (3;4) | (0;0) | (3;5) | (1;5) | (2;3) | (2;3) | (3;4) | (3;3) | (3;5) | (2;3) | (4;4) | (4;4) |
| $C1_3$ | (3;4) | (3;5) | (0;0) | (4;5) | (3;4) | (3;4) | (2;3) | (3;3) | (3;5) | (2;5) | (4;5) | (4;4) |
| $C2_4$ | (3;4) | (5;5) | (5;5) | (0;0) | (2;3) | (2;3) | (3;3) | (3;4) | (4;5) | (3;4) | (3;4) | (5;5) |
| $C2_5$ | (4;5) | (3;5) | (3;4) | (3;4) | (0;0) | (2;3) | (3;5) | (3;5) | (4;4) | (1;4) | (3;4) | (4;4) |
| $C3_6$ | (4;4) | (4;4) | (3;4) | (2;4) | (1;4) | (0;0) | (2;3) | (2;5) | (5;5) | (2;4) | (4;4) | (3;4) |
| $C3_7$ | (4;4) | (5;4) | (4;4) | (2;4) | (2;4) | (2;4) | (0;0) | (4;5) | (2;5) | (2;3) | (3;3) | (3;4) |
| $C4_8$ | (3;3) | (4;4) | (5;5) | (1;4) | (1;2) | (4;5) | (2;3) | (0;0) | (2;5) | (1;1) | (3;5) | (3;4) |
| $C4_9$ | (3;3) | (2;3) | (4;5) | (1;5) | (2;2) | (3;4) | (4;5) | (2;3) | (0;0) | (2;2) | (3;5) | (3;3) |
| $C5_{10}$ | (4;4) | (2;3) | (2;5) | (5;5) | (1;1) | (2;3) | (4;5) | (2;3) | (2;3) | (0;0) | (3;4) | (3;4) |
| $C5_{11}$ | (2;2) | (1;5) | (2;3) | (2;5) | (1;1) | (3;4) | (3;3) | (3;5) | (2;5) | (1;2) | (0;0) | (5;5) |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C5$_{12}$ | (3;3) | (2;5) | (1;3) | (2;3) | (4;5) | (4;4) | (3;3) | (3;3) | (3;5) | (4;5) | (4;4) | (0;0) |

...

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| | C1$_1$ | C1$_2$ | C1$_3$ | C2$_4$ | C2$_5$ | C3$_6$ | C3$_7$ | C4$_8$ | C4$_9$ | C5$_{10}$ | C5$_{11}$ | C5$_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C1$_1$ | (0;0) | (2;5) | (2;5) | (1;5) | (1;5) | (2;4) | (2;5) | (1;5) | (1;4) | (2;2) | (3;4) | (3;3) |
| C1$_2$ | (2;4) | (0;0) | (3;4) | (1;4) | (1;5) | (2;5) | (4;4) | (2;4) | (2;4) | (2;2) | (3;5) | (3;5) |
| C1$_3$ | (1;4) | (3;3) | (0;0) | (3;4) | (3;5) | (3;5) | (2;4) | (2;4) | (2;3) | (3;4) | (4;4) | (3;5) |
| C2$_4$ | (2;4) | (4;4) | (4;4) | (0;0) | (2;5) | (2;4) | (3;4) | (4;4) | (4;4) | (2;5) | (2;5) | (4;5) |
| C2$_5$ | (3;5) | (3;3) | (3;5) | (2;5) | (0;0) | (1;5) | (4;4) | (4;4) | (3;5) | (2;5) | (3;5) | (3;5) |
| C3$_6$ | (2;4) | (4;3) | (3;5) | (1;5) | (1;5) | (0;0) | (2;4) | (1;4) | (4;4) | (3;4) | (4;5) | (3;5) |
| C3$_7$ | (2;4) | (4;3) | (4;5) | (2;5) | (2;5) | (2;3) | (0;0) | (3;4) | (1;4) | (3;4) | (3;4) | (4;4) |
| C4$_8$ | (2;4) | (4;5) | (4;4) | (1;5) | (1;4) | (4;4) | (1;5) | (0;0) | (1;4) | (2;2) | (4;4) | (4;4) |
| C4$_9$ | (1;3) | (1;2) | (4;4) | (1;4) | (2;4) | (2;5) | (3;4) | (1;4) | (0;0) | (3;3) | (2;3) | (4;5) |
| C5$_{10}$ | (3;5) | (1;2) | (1;4) | (4;4) | (1;3) | (1;4) | (4;4) | (1;5) | (1;4) | (0;0) | (4;4) | (4;4) |
| C5$_{11}$ | (1;2) | (1;3) | (1;4) | (1;4) | (1;3) | (4;4) | (2;4) | (4;5) | (1;4) | (2;2) | (0;0) | (3;3) |
| C5$_{12}$ | (1;2) | (2;4) | (1;4) | (1;4) | (4;4) | (4;5) | (2;5) | (5;5) | (4;5) | (3;4) | (4;5) | (0;0) |

Based on the evaluation matrices (Table 3 and Table 1S) it can be noticed that *i* and *j* values differ, meaning that the experts expressed uncertainty when defining the influences of these criteria in the course of the evaluation. In accordance with the procedure governing implementation of the IR-DEMATEL model, the initial comparison matrices in pairwise clusters/criteria were transformed into interval rough matrices. Thus, eight interval rough cluster and criteria matrices, equations (33) and (34), were obtained. Since the interval rough number is composed of two rough sequences (11) and (12) that form the IRN (13), we show the formation of individual rough sequences for a singe position in the criterion matrices (Table 3). Determining the interval rough comparison matrix elements $X^1$, $X^2$, …,$X^8$ is shown using the example of obtaining the elements in position C1$_1$-C1$_2$.

The interval rough number (13) is composed of two rough matrices (11) and (12). For each matrix $X^m$ two rough sequences in position C1$_1$-C1$_2$ that constitute the interval rough number (13) are obtained. Two classes of objects $x^{e}_{ij}$ and $x^{e'}_{ij}$ are chosen from the comparison matrices (Table 3) for the position C1$_1$-C1$_2$. Each class includes eight elements, as stated below:

$$x^{e}_{1_1-1_2}=\{2;3;3;2;3;3;2;2\}$$

$$z^{e'}_{1_1-1_2}=\{3;4;4;5;3;5;4;4\}$$

By applying equations (1) through (8), rough sequences (11) and (12) are formed for every object class. For the first class, we obtain:

$$\underline{Lim}(2)=2,\ \overline{Lim}(2)=\frac{1}{8}(2+3+3+2+3+3+2+2)=2.5;$$

$$\underline{Lim}(3)=\frac{1}{8}(2+3+3+2+3+3+2+2)=2.5,\ \overline{Lim}(3)=3;$$

...

$$\underline{Lim}(2)=2,\ \overline{Lim}(2)=\frac{1}{8}(2+3+3+2+3+3+2+2)=2.5;$$

For the second object class:

$$\underline{Lim}(3)=3,\ \overline{Lim}(3)=\frac{1}{8}(3+4+4+5+3+5+4+4)=4;$$

$$\underline{Lim}(4)=\frac{1}{6}(3+4+4+3+4+4)=3.67,\ \overline{Lim}(4)=\frac{1}{6}(4+4+5+5+4+4)=4.33;$$

...

$$\underline{Lim}(4)=\frac{1}{6}(3+4+4+3+4+4)=3.67,\ \overline{Lim}(4)=\frac{1}{6}(4+4+5+5+4+4)=4.33$$

In this way, the rough sequences that constitute interval rough number are obtained:

$$RN(x^{1L}_{1_1-1_2})=[2.5,3];\ \ RN(x^{1'U}_{1_1-1_2})=[3,4]\ \ \rightarrow\ IRN(z^{1}_{1_1-1_2})=\left([2.5,3],[3,4]\right);$$

$$RN(x^{2L}_{1_1-1_2})=[2,2.5];\ \ RN(x^{2'U}_{1_1-1_2})=[3.67,4.33]\ \ \rightarrow\ IRN(z^{2}_{1_1-1_2})=\left([2,2.5],[3.67,4.33]\right);$$

...

$RN(x_{1_1-1_2}^{8L}) = [2, 2.5]$; $RN(x_{1_1-1_2}^{8'U}) = [3.67, 4.33]$ $\rightarrow$ $IRN(z_{1_1-1_2}^{8}) = ([2, 2.5], [3.67, 4.33])$. The interval rough numbers for the other comparison matrices for pairs of criteria (Table 4) and clusters (Table 2S) are obtained by applying a similar method.

Table 4. Interval rough comparison matrices for pairs of criteria

DM1

| | $C1_1$ | $C1_2$ | $C1_3$ | $C2_4$ | $C2_5$ | ... | $C5_{12}$ |
|---|---|---|---|---|---|---|---|
| $C1_1$ | [(0.0,0.0),(0.0,0.0)] | [(2.5,3),(3.67,4.33)] | [(2.5,3),(3,3.75)] | [(1.5,2),(3,3.75)] | [(1.5,2),(4.25,5)] | | [(3,4),(3.67,4.33)] |
| $C1_2$ | [(3,4),(4,3.75)] | [(0.0,0.0),(0.0,0.0)] | [(3,3.5),(3.75,5)] | [(1,1.25),(3.75,5)] | [(1.25,2),(3,4)] | | [(3.75,4),(3.5,4.67)] |
| $C1_3$ | [(3,5),(4.25,5)] | [(3,3.5),(3,5)] | [(0.0,0.0),(0.0,0.0)] | [(3.25,4),(4.25,5)] | [(3,3.25),(3,4.25)] | | [(3.25,4),(3.67,4.33)] |
| $C2_4$ | [(3,4),(3,3.5)] | [(4.75,5),(4,5)] | [(4.75,5),(4.25,5)] | [(0.0,0.0),(0.0,0.0)] | [(2,2.25),(3.5,4.67)] | | [(4.75,5),(4.75,5)] |
| $C2_5$ | [(4,5),(3,4.5)] | [(3,3.25),(3.5,5)] | [(3,3.25),(3.67,4.33)] | [(2.25,3),(4,4.5)] | [(0.0,0.0),(0.0,0.0)] | | [(3.67,4.33),(4,4.5)] |
| $C3_6$ | [(3.5,5),(3.75,4)] | [(4,4.25),(3,4)] | [(3,3.25),(4,4.5)] | [(1.25,2),(3.67,4.33)] | [(1,1.25),(3,4.25)] | | [(3,3.75),(4,4.5)] |
| $C3_7$ | [(3.25,4),(4.25,5)] | [(4.25,5),(2.75,4)] | [(4,4.25),(3.33,4.33)] | [(0,2),(4,4.5)] | [(2,2),(3,4)] | ... | [(3,3.5),(4,4.25)] |
| $C4_8$ | [(3,4),(3.5,4)] | [(4,4.5),(3.67,4.33)] | [(4.25,5),(3.5,5)] | [(1,1),(3.33,4.33)] | [(1,1),(2.5,4)] | | [(3,4),(3.75,4)] |
| $C4_9$ | [(2.75,4),(2.75,3)] | [(1.25,2),(2,3)] | [(4,4.5),(4.5,5)] | [(1,1.5),(3.5,5)] | [(1.5,2),(2.5,4)] | | [(3,4),(3,4.25)] |
| $C5_{10}$ | [(3.67,4.33),(4.5,5)] | [(1.5,2),(2.25,3)] | [(1.25,2),(3.5,5)] | [(4.25,5),(4.5,5)] | [(1,1.25),(2.67,4.5)] | | [(3,3.25),(3.5,4)] |
| $C5_{11}$ | [(2,3),(1,1.5)] | [(1,1.25),(2.75,5)] | [(1.5,2),(2.33,3.5)] | [(1.25,2),(3.5,5)] | [(1,1),(2.33,3.33)] | | [(3.25,5),(3.5,5)] |
| $C5_{12}$ | [(2.25,3),(1.67,2.33)] | [(2,2.5),(4.25,5)] | [(1,1.25),(3,3.25)] | [(1.5,2),(2.33,3.5)] | [(3.75,4),(4,4.5)] | | [(0.0,0.0),(0.0,0.0)] |

…

DM8

| | $C1_1$ | $C1_2$ | $C1_3$ | $C2_4$ | $C2_5$ | ... | $C5_{12}$ |
|---|---|---|---|---|---|---|---|
| $C1_1$ | [(0.0,0.0),(0.0,0.0)] | [(2,2.5),(4,5)] | [(2,2.5),(3.75,5)] | [(1,1.5),(3.75,5)] | [(1,1.5),(3.5,4.67)] | | [(2.67,3.33),(3,4)] |
| $C1_2$ | [(2.67,3.3),(3,3.75)] | [(0.0,0.0),(0.0,0.0)] | [(3,3.5),(3.33,4.33)] | [(1,1.25),(3.33,4.33)] | [(1,1.25),(4,5)] | | [(3,3.75),(4.25,5)] |
| $C1_3$ | [(2.3,3.67),(4,4.25)] | [(3,3.5),(2.33,3.67)] | [(0.0,0.0),(0.0,0.0)] | [(3,3.25),(3.5,4.67)] | [(3,3.25),(4.25,5)] | | [(3,3.25),(4,5)] |
| $C2_4$ | [(2.67,3.33),(3,3.5)] | [(4,4.75),(3.67,4.33)] | [(4,4.75),(3.5,4.67)] | [(0.0,0.0),(0.0,0.0)] | [(2,2.25),(4.25,5)] | | [(4,4.75),(4.75,5)] |
| $C2_5$ | [(3.67,4.33),(4.5,5)] | [(3,3.25),(2.5,4)] | [(3.0,3.25),(4.0,5.0)] | [(2,2.25),(4.5,5)] | [(0.0,0.0),(0.0,0.0)] | | [(3,4),(4.5,5)] |
| $C3_6$ | [(2.5,4.0),(3,3.75)] | [(4,4.25),(2.67,3.33)] | [(3.0,3.25),(4.5,5.0)] | [(1,1.25),(4,5)] | [(1,1.25),(4.25,5)] | | [(3,3.75),(4.5,5)] |
| $C3_7$ | [(2.5,3.67),(4,4.25)] | [(4,4.25),(2.33,3.33)] | [(4,4.25),(3.75,5)] | [(2,2),(4.5,5)] | [(2,2),(3.67,4.33)] | ... | [(3.5,4),(4,4.25)] |
| $C4_8$ | [(2.67,3.3),(3,3.5)] | [(4,4.5),(4,5)] | [(4,4.25),(3,4.5)] | [(1,1),(3.75,5)] | [(1,1),(3.5,5)] | | [(3.67,4.33),(3.75,4)] |
| $C4_9$ | [(2.33,3.3),(2,2.75)] | [(1,1.25),(1.67,2.33)] | [(4,4.5),(4,4.5)] | [(1,1.5),(3,4.5)] | [(1.5,2),(3.5,5)] | | [(3.67,4.33),(4.25,5)] |
| $C5_{10}$ | [(4.0,5.0),(4.0,4.5)] | [(1,1.5),(1.5,2.67)] | [(1,1.25),(3,4.5)] | [(4,4.25),(4,4.5)] | [(1,1.25),(3.25,5)] | | [(3.25,4),(3.5,4)] |
| $C5_{11}$ | [(1.67,2.33),(1,1.5)] | [(1,1.25),(2,4)] | [(1,1.5),(2.75,4)] | [(1,1.25),(3,4.5)] | [(1,1),(2.75,4)] | | [(2.67,3.67),(3,3.5)] |
| $C5_{12}$ | [(1.5,2.67),(1.0,2.0)] | [(2,2.5),(3.5,4.67)] | [(1,1.25),(3.25,4)] | [(1,1.5),(2.75,4)] | [(3.75,4),(4.5,5)] | | [(0.0,0.0),(0.0,0.0)] |

Next, the interval rough matrices referring to the responses are aggregated. Based on the clusters and criteria response matrices (Table 4 and 2S), and applying equations (26) and (27) the mean interval rough number $IRN(x_{ij}) = \left[ RN(x_{ij}^{L}), RN(x_{ij}^{'U}) \right]$ is obtained. Therefore, the mean interval rough matrices of average responses for the clusters and criteria are obtained, tables 5 and 3S.

Table 5. Mean interval rough matrix of criteria

| | $C1_1$ | $C1_2$ | $C1_3$ | $C2_4$ | ... | $C5_{12}$ |
|---|---|---|---|---|---|---|
| $C1_1$ | [(0.00,0.00),(0.00,0.00)] | [(2.25,2.75),(3.83,4.67)] | [(2.25,2.75),(3.38,4.38)] | [(1.25,1.75),(3.38,4.38)] | | [(2.83,3.67),(3.33,4.17)] |
| $C1_2$ | [(2.83,3.67),(3.50,3.75)] | [(0.00,0.00),(0.00,0.00)] | [(3.00,3.50),(3.54,4.67)] | [(1.00,1.25),(3.54,4.67)] | | [(3.38,3.88),(3.88,4.83)] |
| $C1_3$ | [(2.67,4.33),(4.13,4.63)] | [(3.00,3.50),(2.67,4.33)] | [(0.00,0.00),(0.00,0.00)] | [(3.13,3.63),(3.88,4.83)] | | [(3.13,3.63),(3.83,4.67)] |
| $C2_4$ | [(2.83,3.67),(3.00,3.50)] | [(4.38,4.88),(3.83,4.67)] | [(4.38,4.88),(3.88,4.83)] | [(0.00,0.00),(0.00,0.00)] | | [(4.38,4.88),(4.75,5.00)] |
| $C2_5$ | [(3.83,4.67),(3.75,4.75)] | [(3.00,3.25),(3.00,4.50)] | [(3.00,3.25),(3.83,4.67)] | [(2.13,2.63),(4.25,4.75)] | | [(3.33,4.17),(4.25,4.75)] |
| $C3_6$ | [(3.00,4.50),(3.38,3.88)] | [(4.00,4.25),(2.83,3.67)] | [(3.00,3.25),(4.25,4.75)] | [(1.13,1.63),(3.83,4.67)] | | [(3.00,3.75),(4.25,4.75)] |
| $C3_7$ | [(2.88,3.83),(4.13,4.63)] | [(4.13,4.63),(2.54,3.67)] | [(4.00,4.25),(3.54,4.67)] | [(1.00,2.00),(4.25,4.75)] | ... | [(3.25,3.75),(4.00,4.25)] |
| $C4_8$ | [(2.83,3.67),(3.25,3.75)] | [(4.00,4.50),(3.83,4.67)] | [(4.13,4.63),(3.25,4.75)] | [(1.00,1.00),(3.54,4.67)] | | [(3.33,4.17),(3.75,4.00)] |
| $C4_9$ | [(2.54,3.67),(2.38,2.88)] | [(1.13,1.63),(1.83,2.67)] | [(4.00,4.50),(4.25,4.75)] | [(1.00,1.50),(3.25,4.75)] | | [(3.33,4.17),(3.63,4.63)] |
| $C5_{10}$ | [(3.83,4.67),(4.25,4.75)] | [(1.25,1.75),(1.88,2.83)] | [(1.13,1.63),(3.25,4.75)] | [(4.13,4.63),(4.25,4.75)] | | [(3.13,3.63),(3.50,4.00)] |
| $C5_{11}$ | [(1.83,2.67),(1.00,1.50)] | [(1.00,1.25),(2.38,4.50)] | [(1.25,1.75),(2.54,3.75)] | [(1.13,1.63),(3.25,4.75)] | | [(2.96,4.33),(3.25,4.25)] |
| $C5_{12}$ | [(1.88,2.83),(1.33,2.17)] | [(2.00,2.50),(3.88,4.83)] | [(1.00,1.25),(3.13,3.63)] | [(1.25,1.75),(2.54,3.75)] | | [(0.00,0.00),(0.00,0.00)] |

The mean elements of the interval rough comparison matrix in pairs of criteria in position $C1_1$ -$C1_4$ are established using equations (26) and (27):

$$RN(z_{1_1-2_4}^{L}) = RN(x_{1_1-2_4}^{1L}, x_{1_1-2_4}^{2L}, ..., x_{1_1-2_4}^{8L}) = \begin{cases} z_{1_1-2_4}^{L} = \frac{1}{8}\sum_{e=1}^{8} x_{1_1-2_4}^{eL} = \frac{1}{8}\left(1.5+1+...+1\right) = 1.25 \\ z_{1_1-2_4}^{U} = \frac{1}{8}\sum_{e=1}^{8} x_{1_1-2_4}^{eU} = \frac{1}{8}\left(2+2+...+1.5\right) = 1.75 \end{cases}$$

$$RN(z_{1_1-2_4}^{'U}) = RN(x_{1_1-2_4}^{1'L}, x_{1_1-2_4}^{2'L}, ..., x_{1_1-2_4}^{8'L}) = \begin{cases} z_{1_1-2_4}^{'L} = \frac{1}{8}\sum_{e=1}^{8} x_{1_1-2_4}^{e'L} = \frac{1}{8}\left(3+2.5+...+3.75\right) = 3.38 \\ z_{1_1-2_4}^{'U} = \frac{1}{8}\sum_{e=1}^{8} x_{1_1-2_4}^{e'U} = \frac{1}{8}\left(3.75+4.5+...+5\right) = 4.38 \end{cases}$$

where $e$ denotes $e$-th expert ($e = 1, 2, ..., 8$), $RN(z_{1_1-2_4}^{L})$ and $RN(z_{1_1-2_4}^{'U})$ denote the lower and upper limits of the interval rough number, respectively. The rough sequences $RN(z_{1_1-2_4}^{L})$ and $RN(z_{1_1-2_4}^{'U})$ denote mean interval rough number $IRN(z_{1_1-2_4}) = [(1.25, 1.75), (3.38, 4.38)]$.

Once the mean matrix of the criteria (Table 5) and clusters (Table 2S) is obtained, the second step of the IR-DEMATEL model is to determine the initial direct-relation matrix. By applying equations (30) through (32), the IR elements of the initial direct-relation matrix criteria (Table 6) and clusters (Table 4S) are calculated.

Table 6. Interval rough initial direct-relation matrix of the criteria

| | $C1_1$ | $C1_2$ | $C1_3$ | $C2_4$ | ... | $C5_{12}$ |
|---|---|---|---|---|---|---|
| $C1_1$ | [(0.00,0.00),(0.00,0.00)] | [(0.07,0.07),(0.09,0.09)] | [(0.07,0.07),(0.08,0.08)] | [(0.03,0.05),(0.08,0.08)] | | [(0.06,0.08),(0.08,0.08)] |
| $C1_2$ | [(0.05,0.09),(0.07,0.07)] | [(0.00,0.00),(0.00,0.00)] | [(0.09,0.09),(0.08,0.09)] | [(0.03,0.08),(0.08,0.09)] | | [(0.10,0.11),(0.08,0.09)] |
| $C1_3$ | [(0.10,0.12),(0.10,0.09)] | [(0.06,0.08),(0.06,0.08)] | [(0.00,0.00),(0.00,0.00)] | [(0.09,0.11),(0.09,0.09)] | | [(0.10,0.11),(0.08,0.09)] |
| $C2_4$ | [(0.06,0.08),(0.07,0.07)] | [(0.10,0.12),(0.10,0.12)] | [(0.10,0.12),(0.10,0.12)] | [(0.00,0.00),(0.00,0.00)] | | [(0.10,0.12),(0.10,0.12)] |
| $C2_5$ | [(0.11,0.12),(0.10,0.09)] | [(0.06,0.08),(0.07,0.09)] | [(0.09,0.08),(0.08,0.09)] | [(0.06,0.08),(0.10,0.09)] | | [(0.10,0.11),(0.08,0.09)] |
| $C3_6$ | [(0.08,0.10),(0.08,0.08)] | [(0.08,0.11),(0.08,0.07)] | [(0.09,0.08),(0.08,0.09)] | [(0.03,0.05),(0.09,0.09)] | | [(0.09,0.10),(0.08,0.09)] |
| $C3_7$ | [(0.08,0.11),(0.10,0.09)] | [(0.08,0.11),(0.06,0.07)] | [(0.10,0.11),(0.09,0.12)] | [(0.03,0.10),(0.10,0.09)] | ... | [(0.09,0.10),(0.09,0.12)] |
| $C4_8$ | [(0.08,0.11),(0.08,0.07)] | [(0.10,0.11),(0.09,0.12)] | [(0.10,0.12),(0.10,0.12)] | [(0.03,0.08),(0.08,0.09)] | | [(0.10,0.11),(0.09,0.12)] |
| $C4_9$ | [(0.06,0.08),(0.06,0.06)] | [(0.04,0.05),(0.04,0.05)] | [(0.08,0.11),(0.10,0.12)] | [(0.03,0.08),(0.08,0.09)] | | [(0.10,0.11),(0.09,0.12)] |
| $C5_{10}$ | [(0.10,0.11),(0.10,0.12)] | [(0.04,0.05),(0.04,0.05)] | [(0.06,0.04),(0.08,0.09)] | [(0.10,0.10),(0.10,0.12)] | | [(0.09,0.09),(0.09,0.12)] |
| $C5_{11}$ | [(0.05,0.07),(0.02,0.03)] | [(0.03,0.03),(0.06,0.09)] | [(0.04,0.04),(0.06,0.07)] | [(0.03,0.05),(0.08,0.09)] | | [(0.09,0.11),(0.08,0.08)] |
| $C5_{12}$ | [(0.04,0.05),(0.03,0.04)] | [(0.06,0.06),(0.09,0.09)] | [(0.03,0.03),(0.07,0.07)] | [(0.03,0.05),(0.06,0.07)] | | [(0.00,0.00),(0.00,0.00)] |

The elements of the initial direct-relation matrix criteria (Table 6) in position $C1_1$ -$C1_4$ are are calculated using equations (30)-(32):

$$IRN(d_{1_1-2_4}) = \frac{IRN(z_{1_1-2_4})}{IRN(s)} = IRN\left(\left[\frac{z_{1_1-2_4}^{L}}{s_{ij}^{L}}, \frac{z_{1_1-2_4}^{U}}{s_{ij}^{U}}\right], \left[\frac{z_{1_1-2_4}^{'L}}{s_{ij}^{'L}}, \frac{z_{1_1-2_4}^{'U}}{s_{ij}^{'U}}\right]\right) = \left[\left(\frac{1.25}{34.54}, \frac{1.75}{38.96}\right), \left(\frac{3.38}{42.92}, \frac{4.38}{51.58}\right)\right] = \left[\left(0.03, 0.05\right), \left(0.08, 0.08\right)\right]$$

$$s_{ij}^{L} = \max\left\{\sum_{j=1}^{n} z_{ij}^{L}\right\} = \max\left\{20.75, 26.58, 31.04, 34.54, 30.67, 29, 29, 27.13, 24.38, 25.25, 20.29, 29.92\right\} = 34.54$$

$$s_{ij}^{U} = \max\left\{\sum_{j=1}^{n} z_{ij}^{U}\right\} = \max\left\{27.25, 31.42, 36.21, 38.96, 36.79, 34.25, 34.21, 32.38, 30.83, 31.04, 26.96, 35.79\right\} = 38.96$$

$$s_{ij}^{'L} = \max\left\{\sum_{j=1}^{n} z_{ij}^{'L}\right\} = \max\left\{37.75, 39.46, 40.33, 40.63, 42.92, 40.50, 39.38, 36.71, 33.17, 35.42, 32.42, 38.96\right\} = 42.92$$

$$s_{ij}^{'U} = \max\left\{\sum_{j=1}^{n} z_{ij}^{'U}\right\} = \max\left\{49.75, 49.46, 49.63, 48.29, 51.58, 48.96, 47.58, 48.83, 45.42, 46.17, 45.17, 46.79\right\} = 51.58$$

The initial direct-relation matrix of clusters/criteria is transformed into a total relation matrix of clusters/criteria (Table 7 and 5S) by applying equations (33) and (34).

Table 7. Total relation matrix of criteria

| | $C1_1$ | $C1_2$ | $C1_3$ | $C2_4$ | ... | $C5_{12}$ |
|---|---|---|---|---|---|---|
| $C1_1$ | [(0.22,0.35),(0.56,0.78)] | [(0.29,0.36),(0.62,0.94)] | [(0.29,0.36),(0.7,1)] | [(0.17,0.24),(0.72,1.02)] | | [(0.34,0.46),(0.76,1.01)] |
| $C1_2$ | [(0.34,0.49),(0.65,0.86)] | [(0.27,0.34),(0.55,0.87)] | [(0.36,0.43),(0.72,1.03)] | [(0.19,0.26),(0.74,1.05)] | | [(0.41,0.53),(0.78,1.04)] |
| $C1_3$ | [(0.39,0.57),(0.68,0.88)] | [(0.41,0.47),(0.62,0.95)] | [(0.33,0.4),(0.66,0.95)] | [(0.28,0.35),(0.77,1.05)] | | [(0.47,0.58),(0.8,1.04)] |
| $C2_4$ | [(0.43,0.6),(0.66,0.85)] | [(0.48,0.55),(0.65,0.94)] | [(0.49,0.56),(0.75,1.01)] | [(0.22,0.29),(0.69,0.95)] | | [(0.55,0.67),(0.83,1.03)] |
| $C2_5$ | [(0.41,0.58),(0.71,0.91)] | [(0.4,0.48),(0.67,0.99)] | [(0.41,0.48),(0.79,1.07)] | [(0.25,0.33),(0.82,1.09)] | | [(0.47,0.61),(0.86,1.08)] |
| $C3_6$ | [(0.37,0.55),(0.67,0.87)] | [(0.4,0.47),(0.64,0.94)] | [(0.39,0.45),(0.76,1.03)] | [(0.21,0.29),(0.77,1.05)] | | [(0.43,0.57),(0.82,1.04)] |
| $C3_7$ | [(0.37,0.54),(0.67,0.85)] | [(0.41,0.49),(0.61,0.9)] | [(0.42,0.48),(0.73,0.99)] | [(0.21,0.3),(0.76,1.01)] | | [(0.44,0.57),(0.79,0.99)] |
| $C4_8$ | [(0.35,0.51),(0.62,0.85)] | [(0.39,0.47),(0.6,0.94)] | [(0.4,0.47),(0.69,1.02)] | [(0.2,0.27),(0.71,1.04)] | | [(0.43,0.56),(0.75,1.02)] |
| $C4_9$ | [(0.32,0.49),(0.56,0.79)] | [(0.29,0.38),(0.52,0.85)] | [(0.37,0.45),(0.65,0.96)] | [(0.19,0.27),(0.65,0.97)] | | [(0.39,0.54),(0.68,0.97)] |

| | | | | | |
|---|---|---|---|---|---|
| $C5_{10}$ | [(0.36,0.51),(0.62,0.83)] | [(0.3,0.38),(0.55,0.86)] | [(0.3,0.38),(0.67,0.97)] | [(0.27,0.34),(0.71,0.98)] | [(0.4,0.52),(0.72,0.97)] |
| $C5_{11}$ | [(0.26,0.42),(0.51,0.76)] | [(0.25,0.33),(0.53,0.88)] | [(0.26,0.34),(0.61,0.94)] | [(0.16,0.24),(0.64,0.97)] | [(0.34,0.48),(0.67,0.96)] |
| $C5_{12}$ | [(0.35,0.52),(0.61,0.79)] | [(0.37,0.44),(0.63,0.9)] | [(0.35,0.42),(0.72,0.96)] | [(0.22,0.3),(0.73,0.98)] | [(0.37,0.49),(0.7,0.9)] |

By summing the elements of the total relation matrix of clusters/criteria by rows, equation (36), and by columns, equation (37), the values of the total direct and indirect effects of criterion *j* on other criteria and other criteria on criterion *j* are obtained. These values together with the threshold value ($\alpha$) of the total relation matrix are used for defining the cause-and-effect relationship diagram. The cause and effect relationship (CER) diagram (Figure 4) is formed to visualize the complicated causal relationship of criteria in a visible structural model.

The elements in matrix $T$ (Table 7 and 5S) with a value higher than the threshold value $\alpha$ will be identified and mapped on the diagram (Figure 4) where the $x$-axis denotes $IRN(R_i+C_i)$, and $y$-axis denotes $IRN(R_i-C_i)$. These values will be used for demonstrating the relationship between two factors. In the course of the demonstration, the arrow denoting the cause-effect membership is directed from the element with a value lower than $\alpha$ towards the element characterized by higher value than $\alpha$.

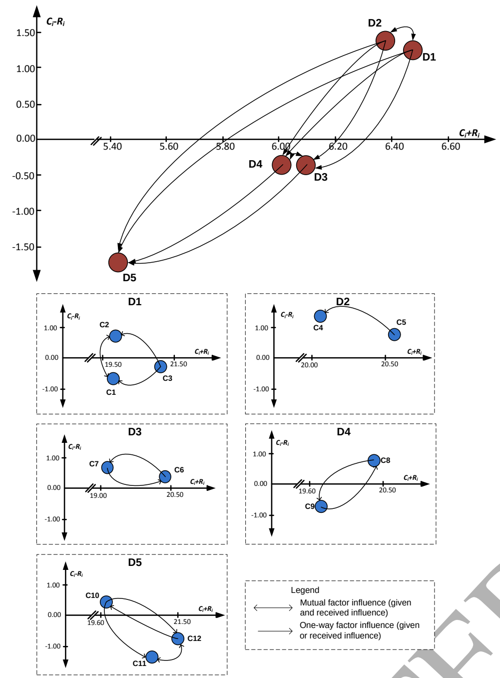


Figure 4. ERD diagram

*Phase 2: The IR-ANP model*

This study applies the IR-ANP method to determine the weights of the 12 criteria and five clusters based on the total relation matrix of the clusters/criteria ($T$) obtained by applying the IR-DEMATEL model (Phase 1). The weight coefficients of the clusters/criteria are formed in Phase 2 based on the CER diagram. The first step in the IR-ANP model is to create a network model based on the CER diagram (Figure 5), while the elements of the interval rough unweighted and weighted supermatrix are calculated based on the total-relation matrix of the clusters/criteria.

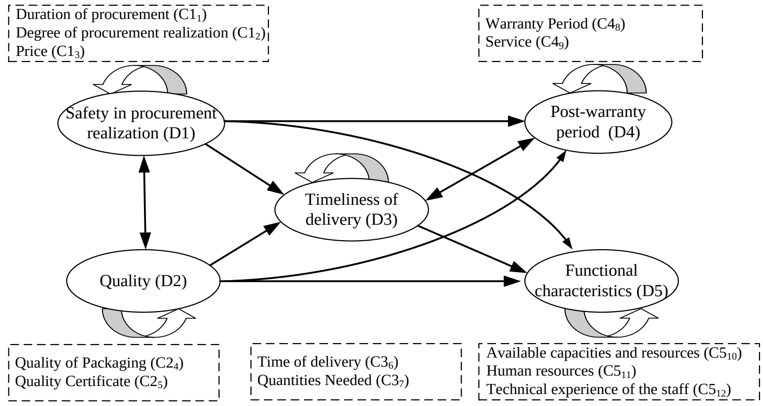


Figure 5. Network model

Here, the total-influence matrix $T$ is included in the ANP model. The unweighted supermatrix and weighted supermatrix are obtained by applying equations (39) through (44). The influential weights of the stable matrix are defined once the unweighted supermatrix and weighted supermatrix are calculated, Table 6S. In the matrix shown in Table 6S each row denotes the weight of a particular criterion. Weights of the clusters/criteria were obtained based on the values specified in Table 6S, as shown in Table 8.

Table 8. Weights of clusters/criteria

| Dimensions/Criteria | Weight coefficient | Rank |
|---|---|---|
| Safety in procurement realization (D1) | [(0.039,0.918),(0.047,1.017)] | 1 |
| Duration of procurement ($C1_1$) | [(0.015,0.289),(0.017,0.331)] | 6 |
| Degree of procurement realization ($C1_2$) | [(0.013,0.255),(0.018,0.298)] | 10 |
| Price ($C1_3$) | [(0.011,0.374),(0.012,0.388)] | 1 |
| Quality (D2) | [(0.104,0.585),(0.023,0.623)] | 3 |
| Quality of Packaging ($C2_4$) | [(0.013,0.287),(0.014,0.308)] | 8 |
| Quality Certificate ($C2_5$) | [(0.090,0.298),(0.010,0.315)] | 3 |
| Timeliness of delivery (D3) | [(0.018,0.502),(0.020,0.578)] | 5 |
| Time of delivery ($C3_6$) | [(0.008,0.285),(0.009,0.311)] | 9 |
| Quantities Needed ($C3_7$) | [(0.010,0.217),(0.011,0.267)] | 12 |
| Post-warranty period (D4) | [(0.018,0.597),(0.017,0.655)] | 4 |
| Warranty Period ($C4_8$) | [(0.090,0.288),(0.011,0.324)] | 7 |
| Service ($C4_9$) | [(0.007,0.309),(0.008,0.331)] | 5 |
| Functional characteristics (D5) | [(0.021,0.935),(0.025,1.031)] | 2 |
| Available capacities and resources ($C5_{10}$) | [(0.008,0.364),(0.009,0.379)] | 2 |
| Human resources ($C5_{11}$) | [(0.003,0.314),(0.005,0.373)] | 4 |
| Technical experience of the staff ($C5_{12}$) | [(0.010,0.257),(0.011,0.279)] | 11 |

In addition to the weight coefficients, Table 8 also includes prioritization of the clusters/criteria. It can be noticed that clusters D1 and D5 are the most influential since they include criteria characterized by the highest weight coefficients, Price ($C1_3$) and Available capacities and resources ($C5_{10}$). These are followed by D3 and D4 clusters. This prioritization of clusters/criteria has confirmed the recommendations on the importance of the criteria suggested by Dobi, et al. (2010) and Chai, et al. (2013).

Since this novel approach has not been widely recognized in the literature, the results (Table 8) were validated by comparing them with the results obtained by traditional approaches such as the crisp DEMTEL-ANP model (Kuo et al., 2015) and fuzzy DEMTEL-ANP model (Pamučar and Ćirović, 2015). Symmetric triangular fuzzy numbers were used when calculating the weight coefficients for the DEMTEL-ANP model. The comparison results are shown in Figure 6, based on which it can be concluded that all three methods generate sequences of weight coefficients characterized by similar ranks ($C1_3 > C5_{10} > C2_5 > C5_{11} > C4_9 > C1_1 > C4_8 > C3_6 > C1_2 > C5_{12} > C3_7$), but various values.

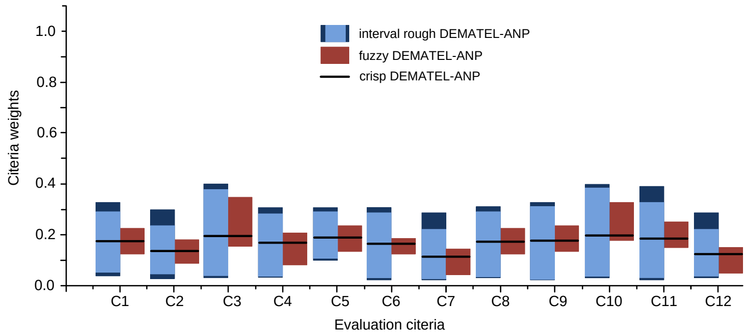


Figure 6. Comparison of the criteria weighting

Figure 6 shows that each interval number is presented using two color shades (dark and light). The darker shade denotes the upper and lower range, while the lighter shade denotes the intersection of two rough sequences of the interval rough number.

The crisp DEMTEL-ANP model calculates the weight coefficients by applying crisp numbers. In this way, uncertainty and vagueness in the group decision making process can be ignored. On the other hand, in the fuzzy DEMTEL-ANP and IR-DEMTEL-ANP models, uncertainties in the group decision making process are represented by various dimensions of fuzzy and rough intervals of weight coefficients. Various interval values are the result of different mechanisms employed for treating uncertainty and subjectivity. While the fuzzy DEMTEL-ANP model deals with uncertainty by means of fuzzy sets with previously defined boundaries that cannot be either extended or narrowed, the IRN boundaries are flexible and can be adjusted to the uncertainties contained in the data. The previously defined boundaries in the fuzzy DEMTEL-ANP model additionally increase subjectivity in the group decision making process since the boundaries are defined based on subjective assessment. This can significantly affect the degree of uncertainty, which is expressed in the interval size, unlike in the IR-DEMTEL-ANP model. In this way, the proposed IR-DEMTEL-ANP model can efficiently measure uncertainties in the course of evaluating criteria and reflect the perception of a decision maker. The results of the weight criteria are used in *Phase 3* to select the most favorable bidder.

*Phase 3: The IR-MAIRCA method*

The IR-MAIRCA method is applied to evaluate the alternative solutions once the weight coefficients of the criteria are determined. Eight experts participated in the evaluation of 10 bidders who submitted their tenders in the public procurement procedure. As with the IR-DEMATEL model, the experts evaluated the alternative solutions by assigning the relevant values specified on a 1–9 scale: 1 – very low influence; 2 – medium low influence influence; 3 – low influence; ... ; 8 – high influence; 9 – very high influence. If expert *e* cannot decide between two values from the linguistic scale, then both values from the scale are given ( $x_{ij}^{e}; x_{ij}^{e'}$ ). The evaluation results are shown in Table 7S. Once the evaluation process was completed by applying equations (45) through (48) the decisions were aggregated and initial decision making matrix *Y* was obtained, Table 9.

Table 9. Aggregated initial decision-making matrix *Y*

| Alter. | $C1_1$ | $C1_2$ | $C1_3$ | $C2_4$ | … | $C5_{12}$ |
|---|---|---|---|---|---|---|
| A1 | [(4,6),(4.63,6.7)] | [(5.63,7.41),(6.58,8.06)] | [(3.87,6.6),(4.86,7.33)] | [(2.04,4.96),(3.04,5.96)] | | [(2.87,4.93),(3.5,5.9)] |
| A2 | [(8.35,8.95),(8.35,8.95)] | [(7.58,8.63),(8.44,8.89)] | [(5.09,7.75),(5.85,8.28)] | [(7.58,8.63),(8.44,8.89)] | | [(6.93,8.55),(7.8,8.8)] |
| A3 | [(6.2,7.77),(7.19,8.33)] | [(4.36,7.38),(5.31,7.91)] | [(3.7,4.9),(4.7,5.9)] | [(4,6),(5,7)] | | [(7.08,8.37),(8.1,8.94)] |
| A4 | [(6.49,8.37),(6.55,8.61)] | [(5.81,8.38),(6.54,8.77)] | [(5.77,8.22),(6.65,8.71)] | [(7.16,8.73),(7.93,8.88)] | | [(6.8,7.8),(7.8,8.8)] |
| A5 | [(6.62,7.82),(7.35,8.38)] | [(5.86,7.67),(6.89,8.39)] | [(5.21,6.63),(5.75,7.42)] | [(6.19,7.33),(6.84,8.26)] | | [(5.59,7.1),(6.58,7.85)] |
| A6 | [(4.49,6.37),(4.84,7.28)] | [(3.86,6.33),(4.79,7.26)] | [(4.68,7.4),(5.29,7.98)] | [(2.65,5.09),(3.18,6.02)] | … | [(4.72,6.67),(5.3,7.42)] |
| A7 | [(6.64,7.8),(6.7,8.16)] | [(5.9,7.67),(6.08,8.1)] | [(3.97,6.56),(4.63,6.84)] | [(2.47,5.28),(3,6.12)] | | [(6.78,8.26),(7.8,8.8)] |
| A8 | [(5.85,7.5),(6.11,7.88)] | [(5.78,8.1),(6.22,8.41)] | [(4.71,6.4),(5.47,7.43)] | [(1.91,5.02),(2.3,6.06)] | | [(6.36,8.21),(7.23,8.71)] |
| A9 | [(4.2,5.36),(4.84,6.3)] | [(2.38,5.04),(3.34,5.92)] | [(3.33,5.97),(3.85,6.96)] | [(3.58,6.27),(4.41,7.17)] | | [(2.69,6),(3.01,6.55)] |
| A10 | [(6.17,8.26),(5.81,8.29)] | [(5.87,7.26),(5.97,7.59)] | [(4.58,6.4),(5.47,7.43)] | [(4.29,7.2),(4.86,7.91)] | | [(4.3,7.36),(4.85,8.12)] |

After aggregating the evaluation criteria (Table 9) the preferences were determined according to the selection of alternatives $P_{Ai}$=1/*m*=0.10, where *m* denotes the number of alternatives (bidders). Since in the evaluation procedure, all bidders are equal (no advantage is given to any particular bidder), preferences $P_{Ai}$ for all alternatives are similar, i.e. $P_{A1}=P_{A2}=...=P_{A10}=0.10$. Based on preferences $P_{Ai}$, and by applying equation (52), the theoretical evaluation matrix ($T_p$) rank *1xn* is obtained (Table 10).

Table 10. IR matrix of theoretical evaluation $T_p$

| Criterion | Theoretical evaluations ($t_p$) | Criterion | Theoretical evaluations ($t_p$) |
|---|---|---|---|
| $C1_1$ | [(0.002,0.029),(0.002,0.033)] | $C3_7$ | [(0.001,0.022),(0.001,0.027)] |
| $C1_2$ | [(0.001,0.026),(0.002,0.030)] | $C4_8$ | [(0.001,0.029),(0.001,0.032)] |
| $C1_3$ | [(0.001,0.037),(0.001,0.039)] | $C4_9$ | [(0.001,0.031),(0.001,0.033)] |
| $C2_4$ | [(0.001,0.029),(0.001,0.031)] | $C5_{10}$ | [(0.001,0.036),(0.001,0.038)] |
| $C2_5$ | [(0.009,0.030),(0.001,0.032)] | $C5_{11}$ | [(0.001,0.031),(0.001,0.037)] |
| $C3_6$ | [(0.001,0.029),(0.001,0.031)] | $C5_{12}$ | [(0.001,0.026),(0.001,0.028)] |

By applying equation (52) we obtain element $C1_1$ from Table 10:

$$t_{p1_1} = P_{A1} \cdot IRN(w_1) = 0.10 \cdot \left[(0.015, 0.289), (0.017, 0.331)\right] = \left[(0.002, 0.029), (0.002, 0.033)\right]$$

In order to determine the real evaluation matrix $T_r$ (Table 11), the elements of the theoretical evaluation matrix (Table 10) are multiplied with the normalized elements of the aggregated initial decision matrix (Table 9). The initial decision making aggregated matrix is normalized by applying equations (54) and (55).

Table 11. Real evaluation matrix $T_r$

| Alter. | $C1_1$ | $C1_2$ | $C1_3$ | ... | $C5_{12}$ |
|---|---|---|---|---|---|
| A1 | [(0.000,0.012),(0.000,0.018)] | [(0.001,0.020),(0.001,0.026)] | [(0.000,0.023),(0.000,0.029)] | | [(0.000,0.009),(0.000,0.014)] |
| A2 | [(0.001,0.029),(0.001,0.033)] | [(0.001,0.024),(0.002,0.030)] | [(0.000,0.031),(0.001,0.036)] | | [(0.001,0.024),(0.001,0.027)] |
| A3 | [(0.001,0.022),(0.001,0.029)] | [(0.000,0.020),(0.001,0.025)] | [(0.000,0.011),(0.000,0.018)] | | [(0.001,0.023),(0.001,0.028)] |
| A4 | [(0.001,0.026),(0.001,0.031)] | [(0.001,0.024),(0.001,0.029)] | [(0.000,0.034),(0.001,0.039)] | | [(0.001,0.021),(0.001,0.027)] |
| A5 | [(0.001,0.022),(0.001,0.029)] | [(0.001,0.021),(0.001,0.028)] | [(0.000,0.023),(0.001,0.029)] | | [(0.000,0.018),(0.001,0.023)] |
| A6 | [(0.000,0.014),(0.000,0.022)] | [(0.000,0.015),(0.001,0.022)] | [(0.000,0.028),(0.000,0.033)] | … | [(0.000,0.016),(0.000,0.021)] |
| A7 | [(0.001,0.022),(0.001,0.028)] | [(0.001,0.021),(0.001,0.026)] | [(0.000,0.022),(0.000,0.025)] | | [(0.001,0.023),(0.001,0.027)] |
| A8 | [(0.001,0.020),(0.001,0.026)] | [(0.001,0.022),(0.001,0.028)] | [(0.000,0.021),(0.000,0.030)] | | [(0.001,0.023),(0.001,0.027)] |
| A9 | [(0.000,0.008),(0.000,0.015)] | [(0.000,0.010),(0.000,0.016)] | [(0.000,0.018),(0.000,0.026)] | | [(0.000,0.014),(0.000,0.017)] |
| A10 | [(0.001,0.025),(0.001,0.029)] | [(0.001,0.019),(0.001,0.024)] | [(0.000,0.021),(0.000,0.0309] | | [(0.000,0.019),(0.000,0.024)] |

By applying equation (55), elements A1-$C1_1$ from Table 9 were normalized:

$$y^-_{A1-C1_1} = \min_{C1_1}\left\{y^L_{A1-C1_1}, y'^L_{A1-C1_1}\right\} = \min_{C1_1}\{4.0, 8.35, 6.2, 6.49, 6.62, 4.49, 6.64, 5.85, 4.2, 6.17\} = 4$$

$$y^+_{A1-C1_1} = \max_{C1_1}\left\{y^U_{A1-C1_1}, y'^U_{A1-C1_1}\right\} = \max_{C1_1}\{6.7, 8.95, 8.33, 8.61, 8.38, 7.28, 8.16, 7.88, 6.3, 8.29\} = 8.95$$

$$IRN(y_{A1-C1_1}) = \left(\left[\frac{4-4}{8.95-4}, \frac{6-4}{8.95-4}\right], \left[\frac{4.63-4}{8.95-4}, \frac{6.70-4}{8.95-4}\right]\right) = \left[(0.00, 0.404), (0.127, 0.546)\right]$$

By multiplying the normalized element $IRN(y_{A1-C1_1})$ by $IRN(t_1)$ from Table 10, i.e. by applying equation (54) we obtain the element in position A1-$C1_1$ from Table 11:

$$IRN(t_{rA1-C1_1}) = \left[(0.002, 0.029), (0.002, 0.033)\right] \cdot \left[(0.00, 0.404), (0.127, 0.546)\right] = \left[(0.000, 0.012), (0.000, 0.018)\right]$$

In the next step, elements of the theoretical evaluation matrix ($T_p$) are deducted from the elements of the real evaluation matrix ($T_p$) to obtain the total gap matrix ($G$). By summing the rows of total gap matrix we obtain the total gap for every alternative, equation (61). Based on the values of the total gap between the theoretical and real evaluations, the initial evaluation of the alternatives is carried out, Table 12.

Table 12. Values of the total gaps for the alternatives and their ranking

| Alter. | Alternative gap *IRN($Q_i$)* | Crisp $Q_i$ | Initial rank | $A_{D,1-j}$ | Final rank |
|---|---|---|---|---|---|
| A1 | [(-0.258,0.349),(-0.187,0.388)] | 0.0738 | 9 | 0.623 | 9 |
| A2 | [(-0.342,0.344),(-0.284,0.378)] | 0.0246 | 2 | 0.051 | 1* |
| A3 | [(-0.312,0.346),(-0.244,0.382)] | 0.0440 | 6 | 0.276 | 6 |
| A4 | [(-0.352,0.345),(-0.292,0.378)] | 0.0202 | 1 | 0.000 | 1 |
| A5 | [(-0.329,0.345),(-0.263,0.380)] | 0.0339 | 3 | 0.159 | 3 |
| A6 | [(-0.289,0.349),(-0.218,0.385)] | 0.0576 | 8 | 0.434 | 8 |
| A7 | [(-0.301,0.347),(-0.236,0.380)] | 0.0482 | 7 | 0.325 | 7 |
| A8 | [(-0.312,0.347),(-0.249,0.382)] | 0.0424 | 4 | 0.258 | 4 |
| A9 | [(-0.232,0.351),(-0.166,0.389)] | 0.0862 | 10 | 0.766 | 10 |
| A10 | [(-0.311,0.348),(-0.249,0.385)] | 0.0437 | 5 | 0.273 | 5 |

It is desirable for an alternative to have the smallest possible gap between the theoretical and real evaluations, and therefore the initially best-ranked alternative is the one with the smallest total gap value, i.e., A4. In order to define the total gap of the final ranked alternatives presented by IRN, they are transferred into crisp values by applying equations (62) and (63). The dominance index of the best-ranked alternative in relation to other alternatives is defined by applying equation (64) as shown in (Table 12). If the dominance index $A_{D,1\text{-}j}$ of the best-ranked alternative in relation to all other alternatives is higher than or equal to the dominance threshold $I_D$, as stated in equation (65), then the initial rank will be taken as final. However, if the dominance index $A_{D,1\text{-}j}$ for any other alternative is smaller than $I_D$ we cannot say that the best-ranked alternative has enough advantage and therefore it will be assigned a rank "1*". In our example, the dominance threshold is $I_D$=0.090. Since the dominance index of alternative A4 in relation to alternative A2 (initially the second-ranked alternative) is smaller than $I_D$ we conclude that A4 does not have enough advantage in relation to A2, and thus alternative A2 will be assigned the corrected rank "1*". The other values $A_{D,1\text{-}j}$ are higher than $I_D$ so the initial rank is retained for the other alternatives.

## 6. Discussion of results

There are two parts to the discussion of the results. The first is a comparison of the results with those obtained from the most frequently used multi-criteria decision making methods (MCDM) for bidder evaluation in the public procurement procedure. The literature analysis presented in Table 1 shows that the most frequently used methods are TOPSIS and ELECTRE I (Roy, 1968) and therefore these were used for the comparison of results. In addition to the TOPSIS and ELECTRE I methods, the MABAC (Pamučar and Ćirović, 2015), TODIM (Gomes and Lima, 1992; Gomes and Rangel, 2009) and VIKOR methods were also used for comparison since they give stabile and reliable results (Ağirgün, 2012; Ruzgys et al., 2014; Mahmoodi and Jahromi, 2014; Pamučar and Ćirović, 2015). These methods were modified by applying the fuzzy technique since this is the most frequently applied approach to the treatment of uncertainty.
The second part is a sensitivity analysis of the IR-D'ANP-MAIRCA model through thirty six scenarios. A detailed analysis of both the first and second parts is given below.

### 6.1. Comparing the ranks of the MCDM methods

The ranks obtained by the IR-MAIRCA model were compared to those of the other MCDM methods mentioned. A comparative presentation of the ranks from various MCDM techniques is shown in figure 7.

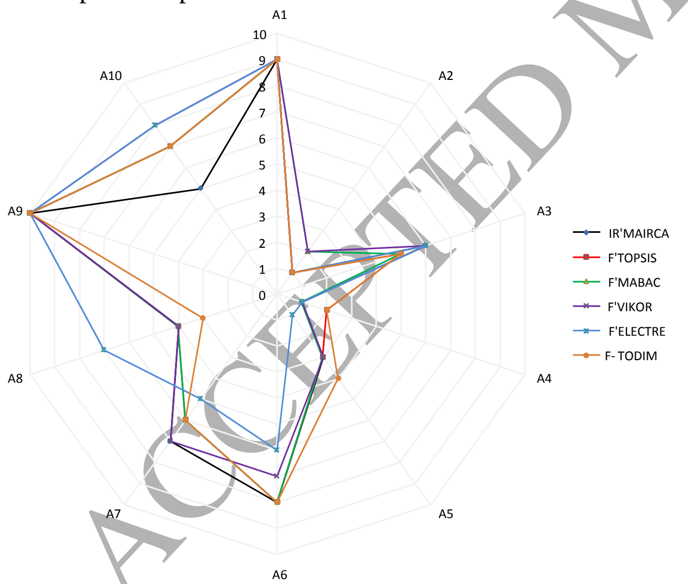


Figure 7. Ranking of alternatives

Since the F-ELECTRE I method does not define the final rank but only the interrelated dominance of alternatives, based on alternative dominance (Figure 8), it obtained the rank A4=A2=A5>A7>A3>A6>A8>A10>A1>A9.

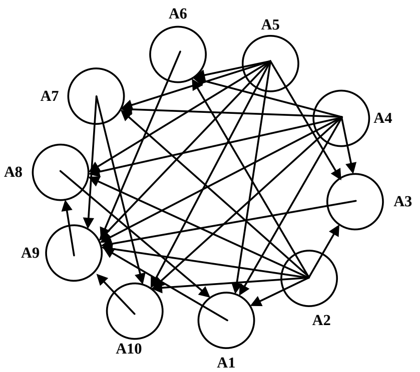

Figure 8. Dominance of alternatives according to the F-ELECTRE I method

Ranking of the alternatives according to the presented methods shows that alternative A4 is best-ranked according to all the methods excluding the F-TOPSIS and F-TODIM methods in which it is second-ranked (A2 is first-ranked). Alternative A2 is also best-ranked according to the IR-MAIRCA method, since alternative A4 is not dominant enough. A2 is also specified as the best-ranked alternative according to the F-TOPSIS and F-ELECTRE I methods, and second-ranked according to the F-VIKOR and F-MABAC methods.

Before making a final decision, the reliability of the results was evaluated in relation to other MCDM techniques. Spearman's rank coefficient of correlation between ranks is one of the most usable and important measuring instruments for determining correlation between the results obtained by various approaches (Ghorabaee et al., 2015). In addition, this coefficient is suitable when dealing with ordinal and/or ranked variables. In this paper, Spearman's coefficient ($r_k$) was used for defining the statistical importance of the difference between the ranks obtained by the IR-MAIRCA model and other approaches. A comparison of the results obtained by applying Spearman's coefficients is shown in Table 13.

Table 13. Rank correlation of the models

| Spearman's coefficient | F-TOPSIS | F-MABAC | F-VIKOR | F-ELECTRE I | F-TODIM | Average value |
|---|---|---|---|---|---|---|
| $r_k$ | 0.958 | 0.958 | 0.933 | 0.818 | 0.945 | 0.922 |

The results show a significant correlation between the ranks of the different MCDM methods. Based on the recommendations of Ghorabaee et al., (2015) all $r_k$ values higher than 0.80 show considerably high correlation. Since in this particular case all $r_k$ values are considerably higher than 0.80, with a mean value 0.922, it can be concluded that there is considerable strong correlation between the proposed approach and the other MCDM techniques tested. Therefore it can be concluded that the proposed rank is confirmed and credible.

### 6.2. Sensitivity analysis

The results of the MCDM methods depend to a great extent on the values of the weight coefficients of the evaluation criteria. Sometimes, ranking of the alternatives may change by modifying the weight coefficients, which will result in change in the sensitivity analysis during the MCDM process. Therefore this section covers a sensitivity analysis of the alternative ranks related to changes in the weight coefficients of the criteria. The sensitivity analysis was carried out through 36 scenarios (Table 14), classified into three phases.

Table 14. Sensitivity analysis scenarios

| Weights of criteria | Ranking |
|---|---|
| $w_{c1}=1.45\times w_{c1(old)}$ | A4>A2>A5>A10>A3>A8>A7>A6>A1>A9 |
| $w_{c2}=1.45\times w_{c2(old)}$ | A4>A2>A5>A8>A3>A10>A7>A6>A1>A9 |
| | ... |
| $w_{c11}=1.45\times w_{c11(old)}$ | A4>A2>A5>A8>A3>A10>A7>A6>A1>A9 |
| $w_{c12}=1.45\times w_{c12(old)}$ | A4>A2>A5>A8>A3>A10>A7>A6>A1>A9 |
| $w_{c1}=1.65\times w_{c1(old)}$ | A2>A4>A5>A10>A3>A8>A7>A6>A1>A9 |
| $w_{c2}=1.65\times w_{c2(old)}$ | A4>A2>A5>A8>A3>A10>A7>A6>A1>A9 |
| | ... |
| $w_{c11}=1.65\times w_{c11(old)}$ | A4>A2>A5>A8>A3>A10>A7>A6>A1>A9 |
| $w_{c12}=1.65\times w_{c12(old)}$ | A4>A2>A5>A8>A3>A7>A10>A6>A1>A9 |
| $w_{c1}=1.85\times w_{c1(old)}$ | A2>A4>A5>A10>A3>A7>A8>A6>A1>A9 |
| $w_{c2}=1.85\times w_{c2(old)}$ | A4>A2>A5>A8>A7>A3>A10>A6>A1>A9 |

| | |
|---|---|
| | ... |
| $w_{c11}$=1.85× $w_{c11(old)}$ | A4>A2>A5>A8>A7>A3>A10>A1>A6>A9 |
| $w_{c12}$=1.85× $w_{c12(old)}$ | A2>A4>A3>A8>A7>A5>A10>A6>A9>A1 |

In Phase 1, the weight coefficients of the criteria in the first twelve scenarios were increased/decreased by 45%. In each of the twelve scenarios, one coefficient with its weight increased by 45 % was favoured. In the same scenario, the weight coefficients of the remaining criteria were decreased by 45 %. In Phase 2, in the next twelve scenarios a similar procedure was applied with the weight coefficients being increased/decreased by 65%. Finally, in phase 3 in twelve scenarios, the weight coefficients were increased/decreased by 85%. Changes in the ranking are shown in Figure 9 and Table 14.

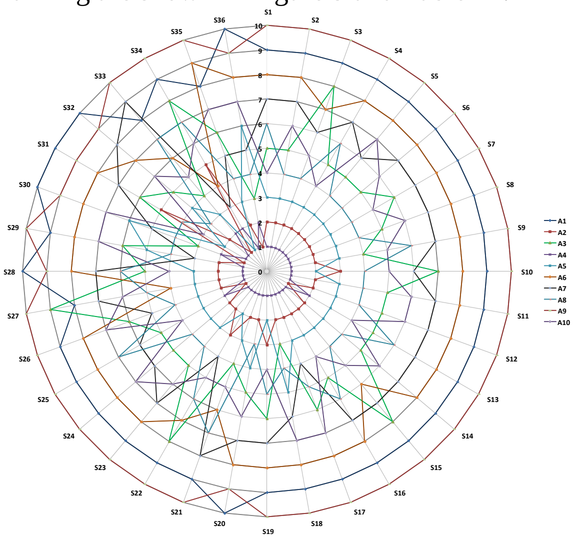


Figure 9. Sensitivity analysis of the alternative ranking through 36 scenarios

The results (Figure 6 and Table 14) show that assigning various weights to the criteria through different scenarios results in a change in the ranks of individual alternatives, thus proving that the model is sensitive to changes in the weight coefficients. Comparison of the best-ranked alternatives (A4 and A2) in scenarios 1 through 36 (results in Table 12) confirmed the ranking of alternatives A4 and A2. Analysis of the ranks through 36 scenarios showed that alternative A4 retained its rank in 30 scenarios (it remained the best-ranked alternative), while in the remaining six scenarios it was second-ranked. The second-ranked alternative A2 retained its rank in 26 scenarios while in 7 scenarios it was the best-ranked alternative. Changing the criteria weights through scenarios resulted in changing the ranks of the remaining alternatives. However, it can be said that these changes were not so drastic, which is confirmed by correlation of the ranks through scenarios (Table 15).

Table 15. Correlation of the ranks through 36 scenarios

| Scenario | $r_k$ | Scenario | $r_k$ | Scenario | $r_k$ |
|---|---|---|---|---|---|
| S1 | 0.958 | S13 | 0.958 | S25 | 0.921 |
| S2 | 0.982 | S14 | 0.982 | S26 | 0.945 |
| S3 | 0.958 | S15 | 0.945 | S27 | 0.824 |
| S4 | 0.958 | S16 | 0.958 | S28 | 0.945 |
| S5 | 0.958 | S17 | 0.909 | S29 | 0.848 |
| S6 | 0.982 | S18 | 0.885 | S30 | 0.824 |
| S7 | 0.994 | S19 | 0.958 | S31 | 0.861 |
| S8 | 0.982 | S20 | 0.958 | S32 | 0.945 |
| S9 | 0.945 | S21 | 0.885 | S33 | 0.861 |
| S10 | 0.958 | S22 | 0.861 | S34 | 0.894 |
| S11 | 0.982 | S23 | 0.982 | S35 | 0.933 |
| S12 | 0.982 | S24 | 0.958 | S36 | 0.824 |

Spearman's correlation coefficients ($r_k$) were obtained by comparing the initial ranks of the IR-D'ANP-MAIRCA model (Table 12) with the ranks obtained through the scenarios. Based on Table 13 it can be noticed that there is significant correlation of the ranks since in 2/3 of the scenarios, the value $r_k$ is higher than 0.909, while in the remaining scenarios it exceeded the value of 0.824. The mean value $r_k$ in all scenarios is 0.922, which is a considerably high correlation. Since all $r_k$ values are considerably higher than 0.80 it can be concluded that there is a considerably high correlation between the ranks and that the proposed rank is both confirmed and credible.

## 7. Conclusion

Respecting uncertainties in the multi-criteria decision making process is a significantly important aspect of both objective and unprejudiced decision making. The multi-criteria decision making process is usually associated with numerous difficulties, since information on multiple attributes needs to be presented using precise numerical values. This is the result of both the complexity and ambiguity of real indicators, as well as imprecision in the human cognitive process. This paper presents a novel approach for treating uncertainty by introducing interval rough numbers. The basic idea of applying algorithms in the decision making process that are based on the interval approach includes the application of interval numbers for presenting attribute values. The advantages of applying IRN are numerous. Interval rough numbers facilitate the decision making process exclusively by using internal knowledge for presenting the decision attributes. In such a way both the subjectivity and assumptions that may affect the attribute value and final selection of alternatives are eliminated. When applying interval rough numbers, only the structure of the given data is used instead of additional/external parameters. Therefore, only uncertainties already contained in data are used, which considerably increases the objectivity of the decision making process. The other advantage of this approach is the suitability of IRN for application in sets dealing with minor data in cases when traditional statistical models are not suitable.

The application of interval rough numbers in the multi-criteria decision making process is presented through a hybrid model composed of the interval rough D'ANP model and the interval rough MAIRCA method. The IR-D'ANP-MAIRCA model is applied to a case study: the evaluation of bidders in the public procurement procedure. The study shows that interval rough numbers can be efficiently applied in multi-criteria decision making models by respecting the uncertainties identified in the decision making process. Another important segment of this paper is the introduction of novel IR-DEMATEL, IR-ANP and IR-MAIRCA models developed by various authors which are a significant contribution to the development of MCDM literature. The proposed models enable the evaluation of alternatives regardless of dilemmas in the decision making process and lack of quantitive information. The results and sensitivity analysis of the IR model show significant stability of the results and point towards successful use of the model in the future.

Since this novel approach is still underrepresented in the literature and in MCDM, future research should be based on the application of IRN in crisp models for determining the weight coefficients of criteria (for example, the interval rough AHP model or the interval rough Best-Worth method,). Further integration of the interval rough approach into existing MCDM models would make a significant contribution to dealing with both uncertainty and subjectivity in the decision making process.

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## Literature

Adil, M., Nunes, M.B., Peng, G.C. (2014). Identifying operational requirements to select suitable decision models for a public sector e-procurement decision support system.*Journal of Information Systems and Technology Management, 11(2),* 211-228.

Ağirgün, B. (2012). Supplier Selection Based on Fuzzy Rough-AHP and VIKOR. *NevşehirÜniversitesi Fen BilimleriEnstitüDergisi*, 2, 1-11.

Amid, A., Ghodsypour, S. H., & O'Brien, C. (2011).A weighted max-min model for fuzzy multi-objective supplier selection in a supply chain.*International Journal of Production Economics, 131(1),* 139–145.

Arce, M.E., Saavedra, A., Míguez, J.L., Granada, E. (2015).The use of grey-based methods in multi-criteria decision analysis for the evaluation of sustainable energy systems.*A review, Renewable and Sustainable Energy Reviews, 47,* 924–932.

Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, pp.87-96.

Aydogan, E.K. (2011). Performance measurement model for Turkish aviation firms using the rough-AHP and TOPSIS methods under fuzzy environment. *Expert Systems with Applications, 38 (4),* 3992–3998.

Azadeh, A., Kokabi, R. (2016) Z-number DEA: A new possibilistic DEA in the context of Z-numbers, Advanced Engineering Informatics, 30, 604–617.

Bajari, P., Lewis, G. (2011). Procurement Contracting with Time Incentives: Theory and Evidence. *Quarterly Journal of Economics, 126,* 1173-1211

Bhattacharya, A., Geraghty, J., & Young, P. (2010). Supplier selection paradigm: An integrated hierarchical QFD methodology under multiple-criteria environment. *Applied Soft Computing Journal, 10(4),* 1013–1027.

Bobar, V., Mandic, K., Delibasic, B., Suknovic, M. (2015). An Integrated Fuzzy Approach to Bidder Selection in Public Procurement: Serbian Government Case Study. *ActaPolytechnicaHungarica,12(12),* 193-211

Büyüközkan, G., &Çifçi, G. (2012). A novel hybrid MCDM approach based on fuzzy DEMATEL, fuzzy ANP and fuzzy TOPSIS to evaluate green suppliers. *Expert Systems with Applications, 39,* 3000–3011.

Chan, F. T. S., & Chan, H. K. (2010). An AHP model for selection of suppliers in the fast changing fashion market. *International Journal of Advanced Manufacturing Technology, 51(9–12),* 1195–1207.

Chou, Y.C., Sun, C.C., & Yen, H.Y. (2012). Evaluating the criteria for human resource for science and technology (HRST) based on an integrated fuzzy AHP and fuzzy DEMATEL approach. *Applied Soft Computing Journal, 12,* 64–71.

Chua, S.J.L., Ali, A.S., Alias, A.B. (2015). Implementation of Analytic Hierarchy Process (AHP) decision making framework for building maintenance procurement selection: Case study of Malaysian public universities. *EksploatacjaiNiezawodnosc –Maintenance and Reliability, 17 (5),* 7–18.

Costantino, N., Dotoli, M., Falagario M., and Fanti, M.P. (2011). Using fuzzy decision making for supplier selection in public procurement. *Journal of public procurement, 11(3),* 403-427

Crispim, J. A., & De Sousa, J. P. (2010). Partner selection in virtual enterprises. *International Journal of Production Research, 48(3),* 683–707.

Dalalah, D., Hayajneh, M., &Batieha, F. (2011). A fuzzy multi-criteria decision making model for supplier selection. *Expert Systems with Applications, 38,* 8384–8391.

Dobi, J. K.. Gugić, D. Kancijan, (2010). AHP As a Decision Support Tool in the Multi criteria Evaluation of Bids in Public Procurement. *Proceedings of the ITI 2010 Int. Conf. on Information Technology Interfaces, 32,* 447- 452.

Duntsch, I., Gediga, G. (1997). The rough set engine GROBIAN. In: A. Sydow (ed.), Proceedings 15th IMACS World Congress, Berlin, Wissenschaft und Technik Verlag, Berlin 4, 613-618.

Ekbatania, M.K., Cats, O., (2015). Multi-Criteria Appraisal of Multi-Modal Urban Public Transport Systems, *18th Euro Working Group on Transportation*, EWGT 2015, 14-16 July 2015, Delft, The Netherlands, 1-11.

European Commission. (2012). A strategy for e-Procurement, MEMO/12/201.

Falagario, M., Sciancalepore, F., Costantino, N., &Pietroforte, R. (2012).Using a DEAcross efficiency approach in public procurement tenders. *European Journal of Operational Research, 218(2),* 523–529.

Fan, Z., Zhang, X., Chen, F., Liu, Y. (2013). Extended TODIM method for hybrid multiple attribute decision making. Knowledge-Based Systems, 42, 40–48.

Gabus, A., &Fontela, E. (1976). *The DEMATEL observer*. Geneva, Switzerland: BattelleGenevaResearchCenter.

Ghorabaee, M.K., Zavadskas, E.K., ZenonasTurskis, Z., Antucheviciene, J. (2016). A new combinative distance-based assessment (CODAS) method for multi-criteria decision-making. *Economic Computation and Economic Cybernetics Studies and Research, 50(3),* 25-44.

Gigović, Lj., Pamučar, D., Bajić, Z., Milićević, M. (2016). The combination of expert judgment and GIS-MAIRCA analysis for the selection of sites for ammunition depot. *Sustainability, 8(4),* 1-30.

Gigović, LJ., Pamučar, D., Božanić, D., Ljubojević, S. (2017). Application of the GIS-DANP-MABAC multi-criteria model for selecting the location of wind farms: A case study of Vojvodina, Serbia. *Renewable Energy, 103,* 501-521.

Gomes, L.F.A.M. and Lima, M.M.P.P. (1992). From Modelling Individual Preferences to Multicriteria Ranking of Discrete Alternatives: A Look at Prospect Theory and the Additive Difference Model. Foundations of Computing and Decision Sciences, 17, pp. 171-184.

Gomes, L.F.A.M. and Rangel, L.A.D. (2009). An Application of the TODIM Method to the Multicriteria Rental Evaluation of Residential Properties. European Journal of Operational Research, 193, pp. 204-211.

Gong, M., Li, H., Jiang, X. (2016). A multi-objective optimization framework for ill-posed inverse problems, CAAI Transactions on Intelligence Technology, 1(3), 225-240.

Govindan K, Kannan D, Shankar M. (2014). Evaluation of Green manufacturing practices using a hybrid MCDM model combining DANP with PROMETHEE. *International Journal of Production Research.53 (21),* 6344-6371.

Guo, J., Zhang, W. (2008). Selection of suppliers based on rough set theory and VIKOR algorithm. *International Symposium on Intelligent Information Technology Application Workshops*, IITAW'08, IEEE, 49–52.

Han H, Li B, Wang K.F., Shi Z (2016) Exploiting structural similarity of log files in fault diagnosis for Web service composition. CAAI Trans Intell Technol 1(1):61–71.

Ishizaka, A., Pearman, C., &Nemery, P. (2012). AHPSort: An AHP-based method for sorting problems. *International Journal of Production Research, 50(17),* 4767–4784.

Ji, W., Tang, L., Li, D., Yang, W., Liao, Q. (2016). Video-based construction vehicles detection and its application in intelligent monitoring system. CAAI Transactions on Intelligence Technology, 1(2), 162-172.

Jin, H., Chen, Q., Chen, Z., Hu, Y., Zhang, J. (2016). Multi-LeapMotion sensor based demonstration for robotic refine tabletop object manipulation task. CAAI Transactions on Intelligence Technology, 1(1), 104-113.

Kahraman, C., Cebeci, U., Ulukan, Z. (2003). Multi-criteria supplier selection using fuzzy AHP. *Logistics Information Management, 16(6),* 382 – 394, DOI 10.1108/09576050310503367

Kang, B. Wei, D. Li, Y. Deng, Y. (2012). A Method of Converting Z-number to Classical Fuzzy Number, Journal of Information & Computational Science, 9 (3), 703-709.

Kang, R., Zhang, T., Tang, H., Zhao, W. (2016). Adaptive Region Boosting method with biased entropy for path planning in changing environment. CAAI Transactions on Intelligence Technology, 1(2), 179-188.

Khoo, L.P., &Zhai, L.Y. (2001). A prototype genetic algorithm enhanced rough set-based rule induction system. *Computers in Industry, 46(1),* 95–106.

Kuang, H, Kilgour, D.M., Hipel, K.W. (2015). Grey-based PROMETHEE II with application to evaluation of source water protection strategies. *Information Sciences, 294,* 376–389.

Kuo, R.J., Hsu, C.W. & Chen, Y.L. (2015). Integration of fuzzy ANP and fuzzy TOPSIS for evaluating carbon performance of suppliers. *International Journal of Environmental Science and Technology 12(5),* 3863-3876. doi:10.1007/s13762-015-0819-9

Kuo, T, Hsu, C, Li, J. (2015). Developing a Green supplier selection model by using a DANP with VIKOR. *Sustainability, 7,* 1661-1689.

Kuo, T.C., Hsu, C.W., Li, J.Y. (2015). Developing a Green Supplier Selection Model by Using the DANP with VIKOR. *Sustainability, 7*, 1661-1689.

Labib, A. W. (2011). A supplier selection model: A comparison of fuzzy logic and the analytic hierarchy process. *International Journal of Production Research, 49(21),* 6287–6299.

Levary, R. R. (2008). Using the analytic hierarchy process to rank foreign suppliers based on supply risks. *Computers and Industrial Engineering, 55(2),* 535–542.

Li,Y., Tang, J., Luo, X.,&Xu, J. (2009). An integratedmethod of rough set, Kano'smodel and AHP for rating customer requirements' final importance. *Expert Systems with Applications, 36(3),* 7045–7053.

Liang, D., Xu, Y., Liu, D. (2017). Three-way decisions with intuitionistic fuzzy decision-theoretic rough sets based on point operators. *Information Sciences, 375,* 183–201.

Liang, H., Ren, J., Gao, Z., Gao, S., Luo, X., Dong, L., Scipioni, A. (2016). Identification of critical success factors for sustainable development of biofuel industry in China based on grey decision-making trial and evaluation laboratory (DEMATEL). *Journal of Cleaner Production, 131,* 500-508.

Lin, R.J. (2013). Using fuzzy DEMATEL to evaluate the green supply chain manage- ment practices. *Journal of Cleaner Production, 40,* 32–39.

Lin, C.J., & Wu, W.W. (2008). A causal analytical method for group decision-making under fuzzy environment. *Expert Systems with Applications, 34,* 205–213.

Liu, H., Tang, H., Xiao, W., Guo, Z.Y., Tian, L., Gao, Y. (2016). Sequential Bag-of-Words model for human action classification, CAAI Transactions on Intelligence Technology, 1(2), 125-136.

Liu, P., & Zhang, X. (2011). Research on the supplier selection of a supply chain based on entropy weight and improved ELECTRE-III method. *International Journal of Production Research, 49(3),* 637–646.

Mafakheri, F., Breton, M., &Ghoniem, A. (2011). Supplier selection-order allocation: A two-stage multiple criteria dynamic programming approach. *International Journal of Production Economics, 132(1),* 52–57.

Martinez L, Liu J, Ruan D, Yang J (2007) Dealing with heterogeneous information in engineering valuation processes. Informstion science, 177(7), 1533–1542.

Mimović, P. Krstić, A. (2016). Application of Multi-Criteria Analysis in the Public Procurement Process. *Optimization Economic Themes. 54 (1),* 103–128.

Nauman, M., Nouman, A., Yao, J.T. (2016). A three-way decision making approach to malware analysis using probabilistic rough sets. *Information Sciences, 374,* 193–209.

Nayagama, V.L.G., Jeevaraja, S., Sivaraman, G. (2016). Complete Ranking of Intuitionistic Fuzzy Numbers. *Fuzzy Information and Engineering, 8 (2),* 237–254.

Nazari, A., Vandadian, S., &Abdirad, H. (2016). Fuzzy AHP Model for Prequalification of Engineering Consultants in the Iranian Public Procurement System. *ASCE Journal of Management in Engineering, 33(2),* 43-54

Ngan, S.C. (2017). A unified representation of intuitionistic fuzzy sets, hesitant fuzzy sets and generalized hesitant fuzzy sets based on their u-maps. *Expert Systems With Applications, 69,* 257–276.

Nguyen, H. (2016). A new interval-value d knowledge measure for interval-valued intuitionistic fuzzy sets and application in decision making. *Expert Systems With Applications, 56,* 143–155.

Pamučar, D., Ćirović,G. (2015). The selection of transport and handling resources in logistics centres using Multi-Attributive Border Approximation area Comparison (MABAC). *Expert Systems with Applications, 42,* 3016- 3028.

Pamučar, D., Vasin, Lj., Lukovac, L. (2014). Selection of railway level crossings for investing in security equipment using hybrid DEMATEL-MARICA model, *XVI International Scientific-expert Conference on Railway*, Railcon, 89-92.

Pawlak, Z. (1982). Rough sets. *International Journal of Computer & Information Sciences, 11(5),* 341–356.

Pawlak, Z. (1991). *Rough sets: Theoretical aspects of reasoning about data* (Vol. 9). Berlin: Springer.

Pawlak, Z. (1993). Anatomy of conflicts. *Bulletin of the European Association for Theoretical Computer Science, 50,* 234-247.

Pourahmad, A., Hosseini, A., Banaitis, A., Nasiri, H., Banaitienė, N., Tzeng, G.-H. (2015). Combination of fuzzy-AHP and DEMATEL-ANP with GIS in a new hybrid MCDM model used for the selection of the best space for leisure in a blighted urban site. Technological and Economic Development of Economy, 21 (5), 773-796.

Qazi, K., Lam, H.K., Xiao, B., Ouyang, G., Yin, X. (2016). Classification of epilepsy using computational intelligence techniques. CAAI Transactions on Intelligence Technology, 1(2), 137-149.

Rezaei, J., Nispeling, T., Sarkis, J., Tavasszy, L. (2016). A supplier selection life cycle approach integrating traditional and environmental criteria using the Best WorstMethod. *Journal of Cleaner Production, 135,* 577–588.

Roy, B. (1968). Classement et choix en présence de points de vue multiples (la méthode ELECTRE). La Revue d'Informatique et de Recherche Opérationelle (RIRO) (8): 57–75.

Roy, J., Chatterjee, K., Bandhopadhyay, A., &Kar, S. (2016). *Evaluation and selection of Medical Tourism sites: A rough AHP based MABAC approach*, doi: arXiv:1606.08962.

Ruzgys, A., Volvačiovas, R., Ignatavičius, Č., Turskis, Z. (2014). Integrated evaluation of external wall insulation in residential buildings using SWARA-TODIM MCDM method. Journal of Civil Engineering and Management, 20(1), 103-110.

Saaty, T. L., Vargas, L. G. (2012). *Models, methods, concepts and applications of the analytic hierarchy process*. Springer Science and Business Media.

Sameh, S., Noufal, K., Abdalah, M. (2016). A fuzzy-AHP multi-criteria decision making model for procurement process. *International Journal of Logistics Systems and Management (IJLSM), 23 (1),* 1-24.

Sen, C. G., Sen, S., &Bas_lgil, H. (2010). Pre-selection of suppliers through an integrated fuzzy analytic hierarchy process and max-min methodology. *International Journal of Production Research, 48(6),* 1603–1625.

Shao, J., Taisch, M., Ortega-Mier, M. (2016). A grey-DEcision-MAking Trial and Evaluation Laboratory (DEMATEL) analysis on the barriers between environmentally friendly products and consumers: practitioners' viewpoints on the European automobile industry. *Journal of Cleaner Production, 112,* 3185-3194.

Shen, K.-Y., Tzeng, G.-H. (2016). Combining DRSA decision-rules with FCA-based DANP evaluation for financial performance improvements. Technological and Economic Development of Economy, 22(5), 685-714.

Shuping, W. (2009). Interval multi-attribute decision-making of attitude indicator method. *Control and decision making, 24 (1),* 35-38.

Sipahi, S., Esen, O. (2010). A multi-criteria model for bidding evaluation: An alternative selection of the best firms for the presentation of Istanbul 2010. *Management Decision 48 (2),* 296–313.

Sizong, G., Tao, S. (2016). Interval-Valued Fuzzy Number and Its Expression Based on Structured Element. *Advances in Intelligent and Soft Computing, 62,* 1417-1425.

Song, W., Ming, X., Han, Y., Wu, Z. (2013a). A rough set approach for evaluating vague customer requirement of industrial product service system. *International Journal of Production Research, 51(22),* 6681–6701.

Song, W., Ming, X., Wu, Z. (2013b). An integrated rough number-based approach to design concept evaluation under subjective environments. *Journal of Engineering Design, 24(5),* 320-341.

Song, W., Ming, X., Wu, Z., & Zhu, B. (2014). A rough TOPSIS approach for failure mode and effects analysis in uncertain environments. *Quality and Reliability Engineering International, 30(4),* 473-486.

Su, C.M., Horng, D.J., Tseng, M.L., Chiu, A.S.F., Wu, K.J., Chen, H.P. (2016). Improving sustainable supply chain management using a novel hierarchical grey-DEMATEL approach. *Journal of Cleaner Production, 134,* 469-481.

Sugihara, K., Maeda, Y., Tanaka, H. (1999). Interval Evaluation by AHP with Rough Set Concept, Lecture Notes in Computer Science, 1711, pp. 375-381

Tiwari, V., Jain, P.K., Tandon, P. (2016). Product design concept evaluation using rough sets and VIKOR method. *Advanced Engineering Informatics, 30,* 16-25.

Topcu, V.I. (2004). A decision model proposal for construction contractor selection in Turkey. *Building and Environment, 39,* 469–481.

Tseng, M. L. (2009). A causal and effect decision making model of service quality ex- pectation using grey-fuzzy DEMATEL approach. *Expert Systems with Applications, 36,* 7738–7748.

Vahdani, B., &Zandieh, M. (2010). Selecting suppliers using a new fuzzy multiple criteria decision model: The fuzzy balancing and ranking method. *International Journal of Production Research, 48(18),* 5307–5326.

Vahdani, B., Tavakkoli-Moghaddam, R., MeysamMousavi, S., Ghodratnama. A. (2013). Soft computing based on new interval-valued fuzzy modified multi-criteria decision-making method. *Applied Soft Computing, 13,* 165–172.

Veselinović, I. (2014). Multi-criteria methods and models for decision making in public procurement. *FACTA UNIVERSITATIS Series: Economics and Organization 11(3),* 261 – 279

Wang, H., Yang, B., Chen, W. (2016). Unknown constrained mechanisms operation based on dynamic interactive control. CAAI Transactions on Intelligence Technology, 1(3), 259-271.

Wang, J., Tang P. (2011). A rough random multiple criteria decisionmaking method based on Interval rough operator. Control and decision making, 26(7) , 1056-1059.

Wang, J., Wang, J.Q, Zhang, H.Y, Chen, X.H. (2015). Multi-criteria decision-making based on hesitant fuzzy linguistic term sets: An outranking approach. *Knowledge-Based Systems, 86,* 224–236.

Xie, G. Zhang, J. Lai, K.K. Yu, L. (2008). Variable precision rough set for group decisionmaking: An application. *International Journal of Approximate Reasoning, 49 (2),* 331–343.

Xu, Z., Wang, H. (2016). Managing multi-granularity linguistic information in qualitative group decision making: an overview. Granular computing, 1(1), 21–25.

Yang, J.L., Tzeng, G.H. (2011). An integrated MCDM technique combined with DEMATEL for a novel cluster-weighted with ANP method. *Expert Systems with Applications, 38,* 1417-1424.

Yang, W., Sun, X., Deng, W., Zhang, C., Liao, Q. (2016). Fourier Locally Linear Soft Constrained MACE for facial landmark localization. CAAI Transactions on Intelligence Technology, 1(3), 241-248.

Yang, Y, Shieh, H, Tzeng, G. (2013). A VIKOR technique based on DEMATEL and ANP for information security risk control assessment. *Information Sciences, 232,* 482-500.

Yu, M., Goh, M., & Lin, H. (2012). Fuzzy multi-objective vendor selection under lean procurement. *European Journal of Operational Research, 219(2),* 305–311.

Zadeh, L.A. (1965). Fuzzy sets, Information and Control, 8(3), pp. 338-353.

Zeshui, X. Qingli, D. (2003). The possibility of interval number sequence method and its application. *Journal of Systems Engineering, 18 (1),* 67-70.

Zhai, L.Y., Khoo, L.P., &Zhong, Z.W. (2008). A rough set enhanced fuzzy approach to quality function deployment. *International Journal of Advanced Manufacturing Technology, 37(5–6),* 613-624.

Zhai, L.Y., Khoo, L.P., &Zhong, Z.W. (2010). Towards a QFD-based expert system: A novel extension to fuzzy QFD methodology using rough set theory. *Expert Systems with Applications, 37(12),* 8888-8896.

Zhang, Q., Xie, Q., Wang, G. (2016). A survey on rough set theory and its applications, CAAI Transactions on Intelligence Technology, 1(4), pp. 323-333.

Zhao, H., and Guo, S. (2014). Selecting Green Supplier of Thermal Power Equipment by Using a Hybrid MCDM Method for Sustainability. *Sustainability, 6(1),* 217-235.

Zheng, P., Xu, X., Xie, S.Q. (2016). A weighted interval rough number based method to determine relative importance ratings of customer requirements in QFD product planning. *Journal of Intelligent Manufacturing, 27, 1-14.*doi 10.1007/s10845-016-1224-z.

Zhong, Q.B., Chen, F. (2016). Trajectory planning for biped robot walking on uneven terrain – Taking stepping as an example. CAAI Transactions on Intelligence Technology, 1(3), 197-209.

Zhou, X., Shi, Y., Deng, X., Deng, Y. (2017). D-DEMATEL: A new method to identify critical success factorsin emergency management. *Safety Science, 91,*93–104.

Zhu, G.N., Hu, J., Qi, J., Gu, C.C., Peng, J.H. (2015). An integrated AHP and VIKOR for design concept evaluation based on rough number. *Advanced Engineering Informatics, 29,* 408–418.

Zywica, P., Stachowiak, A. Wygralak, M. (2016). An algorithmic study of relative cardinalities for interval-valued fuzzy sets. *Fuzzy Sets and Systems, 294,* 105–124.