Serbian Association for Geometry and Graphics

The 5th International Scientific Conference on Geometry and Graphics

moNGeometrija 2016

June 23th - 26th, Belgrade, Serbia

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(alphabetically ordered by the Paper title)

PEDAGOGIC POTENTIAL OF A PARAMETRIC SYSTEM BASED ON THE BOX PACKING CONCEPT

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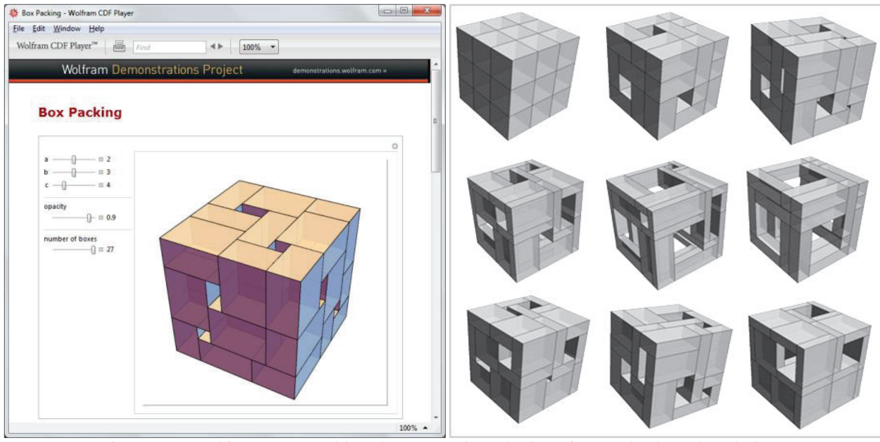
ABSTRACT

In this study we examine pedagogic values of the Box Packing concept, a puzzle problem, initially published as a demonstration within the Wolfram Demonstration Project, gathering 27 identical boxes of any proportion, in a regular cubic body. Methodologically based on the content analysis of realized educational experiments, the research is aimed at enlarging possible use of mathematical/geometric concepts in architectural education, particularly in domain of parametric design. After briefly explaining the concept and original demonstration, and giving an overview of its usage in several courses in various contexts, highlighting pedagogic implications of its use (including development of capabilities to recognize architectonics in pure geometric form, acquiring basic modelling skills, manipulating geometric bodies, understanding parametric systems, materialization in various materials including recycled ones, etc.) we focus on its transposition into a parametric system created as a Grasshopper definition (visual algorithmic editor for Rhino 3D software), analyzing the creation of algorithm and characteristic elements of the definition and discussing their potential application in solving other parametric problems.

Keywords: parametric design; educational methodology; packing problems, algorithm

INTRODUCTION

The research has been initiated several years ago, after discovering a modest, yet very famous book Tooling, by American young architects Aranda and Lash [1], in which seven generative concepts have been examined in creation of architectural form. The selected set of these concepts have been further studied within the Chair for Mathematics, Architectural Geometry and CAAD, at the University of Belgrade, Faculty of Architecture. The Packing concept, as one of them, was recognized as the most expressive and comfortable for implementation in courses of parametric modelling because of its pedagogic potential. Looking for a spatial 3D interpretation of the Packing concept, The Box Packing demonstration has been discovered within the Wolfram Demonstration projects2 (Figure, 1). Contributed by Ed Pegg Jr, this demonstration shows the packing of 27 identical boxes with size $a \times b \times c$ into a cube measuring $a + b + c$ on a side. The architectural and pedagogic potential of the concept has been examined primarily within elective course Generic explorations 06 and integrated in the 3D Visual communications course offered to a group of first year students and continued in all elective courses titled Parametric modelling 1,2 with master students.



Figure, 1 **:** Wolfram Box Packing demonstration, the interface and selected variations

2. USE OF THE BOX PACKING CONCEPT IN VARIOUS CONTEXTS

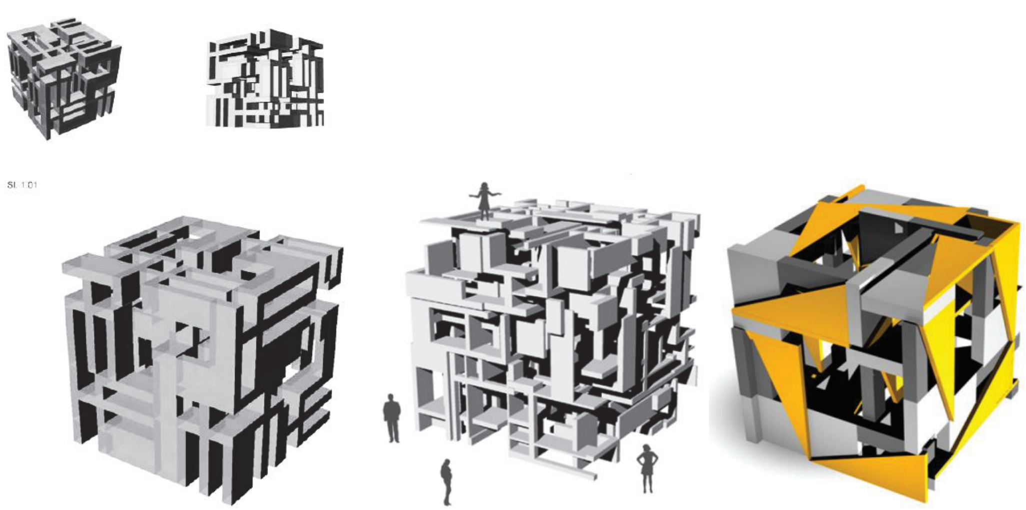
The Box Packing concept described above has been used in numerous different contexts, particularly educational, offered to by the Chair for Mathematics, Architectural Geometry and CAAD at the Faculty of Architecture in Belgrade. Recently it started to be exhibited and promoted to industry, first of all to the regional cluster of natural stone producers.

2.1. Architecture Education

The use of Box Packing in education is especially interesting for this study, focusing on pedagogic potential. We are particularly interested in a range of possible learning outcomes, i.e. what the students could learn dealing with this concept within various courses including: CAAD Principles, Parametric modelling and 3D Visual Communication. The following learning outcomes have been identified so far:

Understanding parameterized geometry - The Wolfram Demonstration of the Box Packing concept is an excellent example of geometric parameterization. Being available online and requiring just a piece of free software for viewing, it is suitable for novices to acquire basic understanding of parameterized structure and a richness of varieties that could be created based on it, as well as to explore its architectonics.

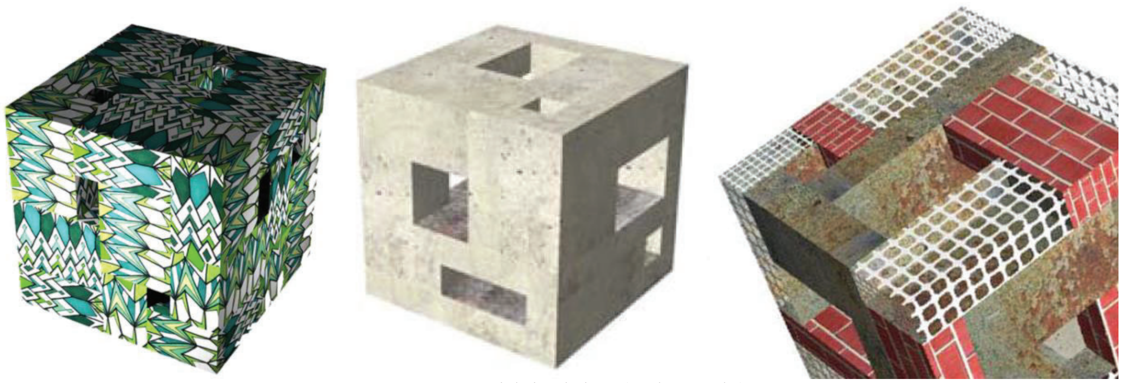
Acquiring basic modelling skills - The Box Packing parametric system has been used as a reference for introducing basic 3D modelling techniques in many CAD software packages, including SketchUp, AutoCAD and Rhino 3D. Consisting of 27 regular boxes of a same size, it is suitable as a task for the very beginning of mastering 3D modelling technique, requiring basic routines like controlling the size of elements, multiplication, moving elements within a virtual 3D space, precise positioning and a spatial rotation (rotation 3D).



Figure, 2: Box Packing variation by replacing the initial element with a more complex one (student works)

Applying advanced modelling - Many of the available 3D modelling software allow creation of predefined complex elements, in AutoCAD and Rhino 3D well known as blocks, in SketchUp as components. Such elements, once defined and multiplied, could be later redefined, affecting all instances within the model. Properly mastered, this simple technique allows replacing simple regular boxes with some irregular or complex elements, enriching significantly the overall design experience (Figure, 2).

Simulating various materials - A model based on the Box Packing concept permits an almost absolute control of mapping, when simulating various materials to the structure. Compared with an architectural model, box packing has limited number of identical elements and therefore permits a careful material simulation for each surface, i.e. deciding about material map size and ratio, map position and orientation, intensity etc. The students particularly liked simulation of wood, concrete and natural stone because of its beautiful yet demanding textures (Figure, 3).



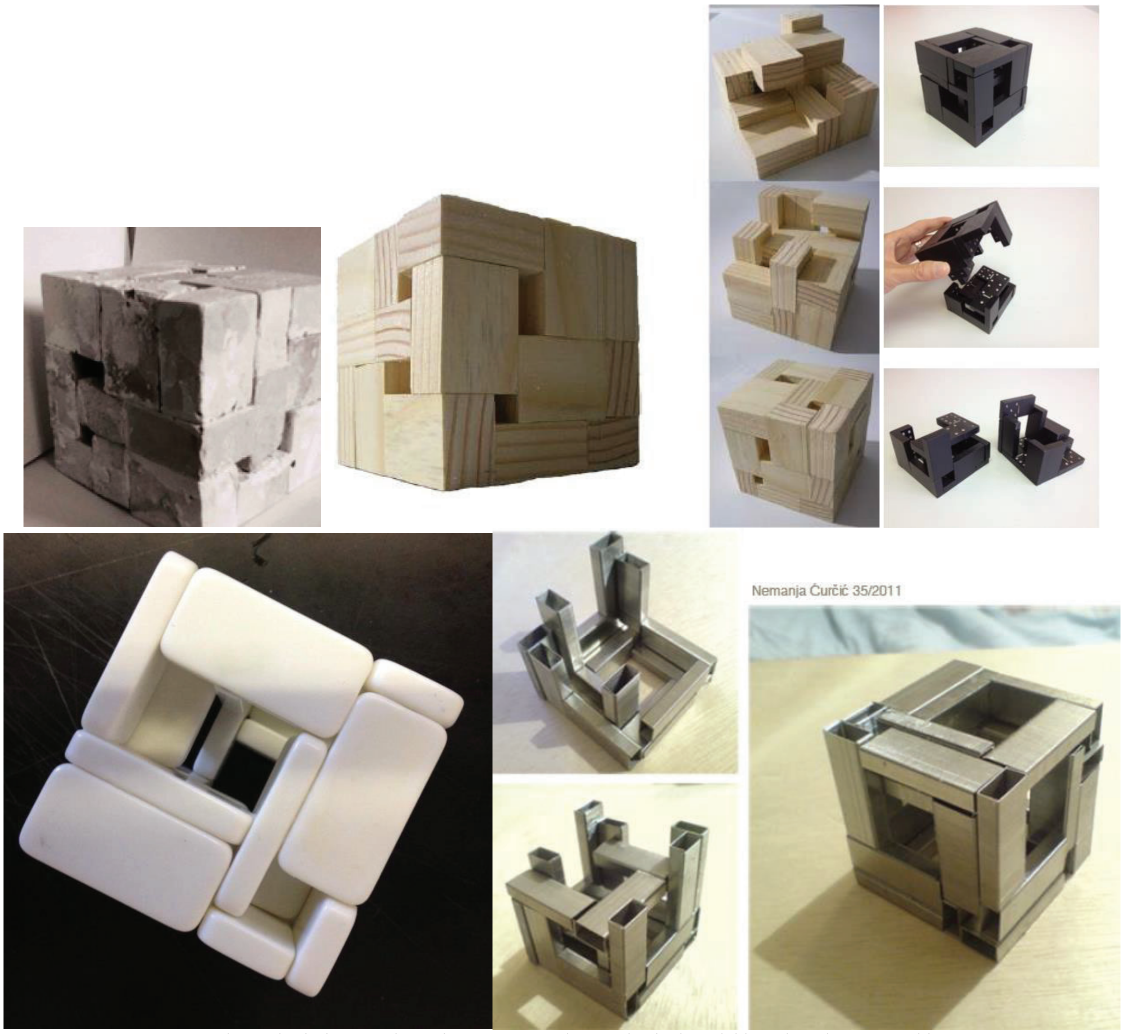
Figure, 3: Material simulations (student works)

Sequencing - Sequence is a way to express potential of a parametric system through a number of variations that clearly represent an idea or intention in change of selected parameter (Figure, 4). Although a parametric system can consist of many parameterized elements, within a sequence of instances it is preferable to vary one or two of available parameters, so that the sequence clearly communicates the character of the system. Mastering the technique of sequencing helps in general understanding and communicating particular parametrics.

Figure, 4: A sequence of structures explaining changes in form caused by tuning the ratio between the basic element edge sizes (student work)

Making physical models - Production of a physical model in the case of Box Packing presumes some challenging precedent strategic decisions, which could affect overall quality of future model, it visual effect, expressiveness, price, invested time, technological demand, etc. (**Figure, 5**).While some students approach the activity by planning production of 27 identical components (cutting wood, stone or plexi-glass, in some cases even sponge), the others are able to identify the appropriate elements in their nearest environment (spared dominos, bricks, match boxes, soap boxes, prismatic candies, etc.).

Dealing with recycled materials – The experience gained in classroom teaching, when students very successfully used variety of cubic elements available in quotidian life for creation of Box Packing structures, shaped an idea of using various recycled materials, following principles of sustainability. An example of the box packing of recycled materials has been made from 27 styropor foam protection boxes remained after new computer lab installation in a new building of the Faculty of Civil Engineering in Rijeka, 2012, during the DAAD funded GEFFA[1] Workshop (**Figure, 6**).



Figure, 5: Student submissions on the assignment to produce a physical model based on the Box Packing concept

Figure, 6: The GEFFA Workshop, Rijeka, 2012; Creation of Box Packing structure made of spared styropor boxes

[1] Geometry Education for Future Architects, a regional cooperation project funded by German Academic Exchange Service in 2012. The Box Packing virtual environment is available at: http://elearning.rcub.bg.ac.rs/moodle/course/view.php?id=181 (accessed April 2016).

2.2. Cooperation with industry

It is an emerging trend in recent decades that advanced geometry results tend to be adopted by the industry of building components and construction. The Box Packing concept has been so far recognized as very interesting for a promotion of various natural materials, mainly the regional natural stone.

Promotion of local natural stone - Association "Stone of Serbia" gathers the cluster of stone professionals on the national level and fosters regional cooperation. The Box Packing based structures have been recognized as suitable for a promotion of local/regional stone. In that terms the students are invited to exhibit their works at several events, including international exhibition "Stone in Art and Architecture 2014" (Belgrade), "Stone Expo 2015" (Kragujevac), etc.

Stone-art Colony "PrilepskiKrinovi 2016", Krin KG Company, Prilep, Macedonia–Within an installation titled "Time Journey" a narrative potential of the Box Packing geometry materialized in natural stone has been examined (**Figure, 7**).

Figure, 7: Art Colony 'Prilepski Krinovi 2015": Installation „Time jorney" by M. Devetaković and Z. Đajić, examining narrative potential of the Box Packing geometry materialized in natural stone from the Balkans region

Exhibiting sustainable building materials - The most recent example of a Box Packing has been completed at the time of concluding this paper and exhibited at SEEBE – South East Europe Belgrade Building Expo. This installation consists of 27 plates of natural stone – granite, marble and travertine. The size of plates is 80x30x6cm, which makes the box packing cube 126x126x126cm. It has been rotated so that its diagonal takes a vertical position, so that its height with a marble post reaches almost 2 meters and its weight exceeds 1 ton (Figure, 8).

Figure, 8: Construstion of the Box Packing structure of natural stone and the final installation at the Belgrade Stone Expo 2016

3. ANALYSIS AND REALISATION OF ALGORITHM AND ITS COMBINATORIAL PREFERENCES

Packing is a mathematical concept with powerful organizational method in which a packed element's position in regard to its neighbours is determined by certain rules – close, but no overlapping. Generally, this concept encourages a sense of democracy where one element's inclusion implies either an understanding of every other element or possibly a readjustment of the entire population, so can be observed as a collective and emergent sense of space – close, but not too close. The Box Packing concept is a topological model where identity and position of each of the elements or parts of it within the system are determined exclusively through its relation with all other elements within the system.

Let's start the algorithm by an overall view to its flow of information and input parameters. There are 27 identical cubes with *a*, *b*, *c* edges which should be puzzled into the cubic frame measuring

$$a+b+c \qquad \textbf{(Eq.1)}$$

The values *a*, *b* and *c* are input parameters which we want to vary. If these variables have the same value, then we get specific trivial case which is well known as Magic Hungarian Rubik's Cube (Figure, 9).

Figure, 9 : Special case $a = b = c$

The frame is divided into three floors or levels with nine elements. To work generally with different values of parameters a , b and c , design process begins from the lowest first layer and then other two layers will be added. There are nine identical elements on each layer and the first nine of them should be packed first. First question in solving the problem is how mach different ways of regular packing can be made at the first layer and the easiest answer comes from combinatorial graphic matrix (Figure, 10) of first layer when first element is packed lying on its defined a edges on the base of frame.

a	b	c	a	c	b
b	c	a	b	a	c
c	a	b	c	b	a
a	b	c	a	c	b
c	a	b	c	b	a
b	c	a	b	a	c

Figure, 10 : Arrangements for the lowest layer implied by one packed element

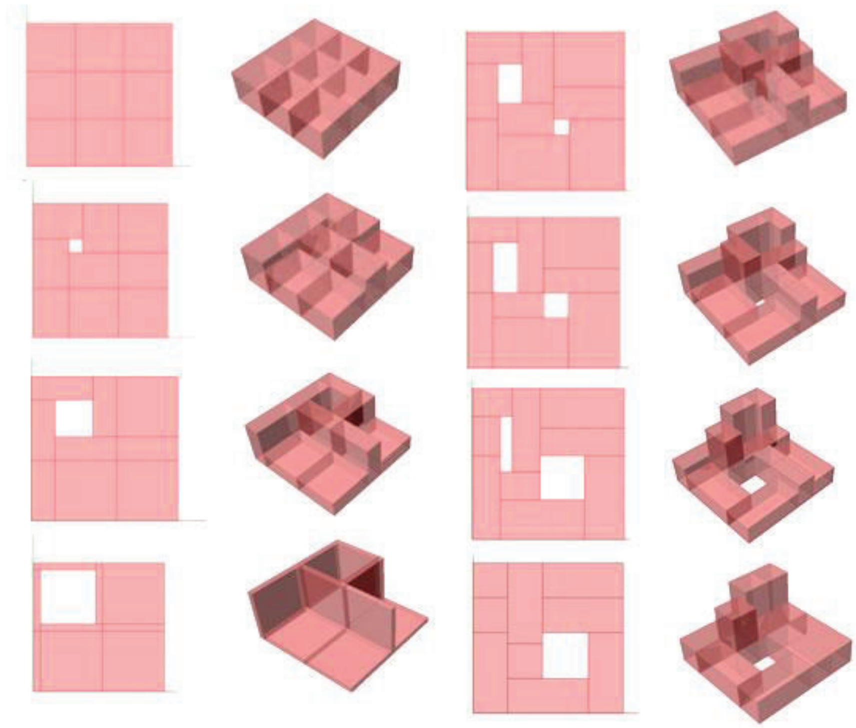
Eq. 1 implies the specific position of packed element – at the one edge of frame must be packed three different oriented elements. Underlined arrangements are uniquely defined by the others implying 12 permutations in total.

At this point, we can't be sure how much non-isomorphic final packed solution can be got from twelve different first layers and leave this question for later analysis. Methodologically, we intend to find just one correct solution and further explore the others and finally make geometrical and visual comparison if solutions aren't isomorphic. This pedagogical manner enables us to teach the students how to approach and realize specific combinatorial geometrical problem.

In order to realize chosen arbitrary combination of first layer graphic matrix and the main condition of packing – no overlapping elements, we suppose conditional relation

$$a \le b \le c \qquad \textbf{(Eq.2)}$$

Some of different configurations of the first layer are explored by decreasing the value of variable a and increasing the value of variable c (Figure, 11). It has been noticed that when $a = b = c$ there isn't any hole, when $a = b$ or $b = c$ there is just one hole and in another cases, there are two holes on the base of frame. Size of holes is directly implied by proportion of parameters a, b and c.



Figure, 11 - Decreasing of maximal and increasing of minimal parameters

Table 1: Corner's coordinates of all packed elements

1	*(0,0,0)*		*(a,c,b)*
2	*(a,0,0)*		*(a+c,b,a)*
3	*(a+c,0,0)*		*(a+b+c,c,a)*
4	*(0,c,0)*		*(a,b+c,c)*
5	*(a,b,0)*		*(a+b,a+b,c)*
6	*(a+b,c,0)*		*(a+b+c,a+c,b)*
7	*(0,b+c,0)*		*(b, a+b+c,c)*
8	*(b,a+b,0)*		*(a+b, a+b+c,b)*
9	*(a+b,a+c,0)*		*(a+b+c,a+b+c,a)*
10	*(0,0,b)*		*(b,a,b+c)*
11	*(b,0,a)*		*(b+c,a,a+b)*
12	*(b+c,0,a)*		*(a+b+c,c,a+b)*
13	*(0,a,c)*		*(c,a+b,a+c)*
14	*(c,a,c)*		*(b+c,a+c,a+c)*
15	*(b+c,c,b)*		*(a+b+c,b+c,b+c)*
16	*(0,a+b,c)*		*(b,a+b+c,a+c)*
17	*(b,a+c,b)*		*(a+b,a+b+c,b+c)*
18	*(a+b,b+c,a)*		*(a+b+c,a+b+c,a+b)*
19	*(0,0,b+c)*		*(c,b,a+b+c)*
20	*(c,0,a+b)*		*(b+c,a, a+b+c,)*
21	*(b+c,0,a+b)*		*(a+b+c,b, a+b+c)*
22	*(0,b,a+c)*		*(c,a+b, a+b+c)*
23	*(c,a,a+c)*		*(a+c,a+c, a+b+c)*
24	*(a+c,b,b+c)*		*(a+b+c,b+c,a+b+c)*
25	*(0,a+b,a+c)*		*(a, a+b+c, a+b+c)*
26	*(a,a+c,b+c)*		*(a+c,a+b+c,a+b+c)*
27	*(a+c,b+c,a+b)*		*(a+b+c,a+b+c,a+b+c)*

The next step involves generating the special position coordinates of elements which should be packed on the next level. Obviously, position and dimension of just placed elements define gradually hierarchies at the next layer and this goes on, step by step to produce the whole geometry which we call form. Basically this algorithm of packing deals with the huge amount of data and calculations which grows through the flow of algorithms (Table 1).

These three levels are strong interconnected and their members affect each other and in that sense this method can be called associative. Final packed form is comprised of different hierarchies, each associated with logic and position details of all other preceding.

4. STEPS OF DEVELOPMENT OF GRASSHOPPER DEFINITION – TRANSPOSING MATHEMATICAL CONCEPT INTO PARAMETRIC SYSTEM

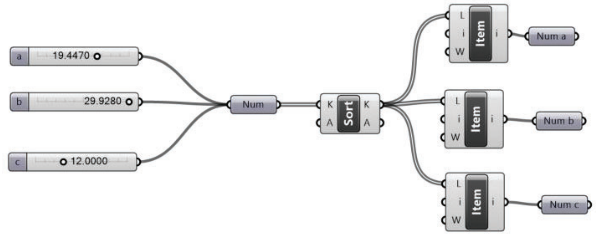
Packing is a mathematical concept with powerful organizational method in which a packed element's position in regard to its neighbours is determined by certain rules – close, but no overlapping. Generally, this concept encourages a sense of democracy where one element's inclusion implies either an understanding of every other element or possibly a readjustment of the entire population, so can be observed. For exploring geometrical, architectural and pedagogical potential of described concept, we worked on transposing Wolfram demonstration into some CAD or parametric software tool and chose Rhinoceros with its plug-in Grasshopper. Rhinoceros (Rhino) is a stand-alone, commercial NURBS-based 3D modelling tool, developed by Robert McNeel & Associates [6], commonly used for design, architecture, CAD/CAM, rapid prototyping, reverse engineering as well as multimedia and graphic design industry. Increasing popularity of this software is based on its diversity, multi-disciplinary functions, low-learning curve, relatively low cost, rich and powerful features set and its ability to import and export over 30 files formats, which allows Rhino to act as a converter tool between programs in design workflow. Popular among students and professionals, Rhinoceros modelling tool is endemic in the architectural design world.

It gained its popularity and predominant role in architectural design because of the Grasshopper plug-in for computational parametric design. Grasshopper is a visual programming language; graphical algorithm editor tightly integrated with Rhino's 3D modelling tools and presents one of the most powerful parametric platforms. This tool requires no knowledge of programming or scripting, but still allows designers and students to build form generators from the simple to the awe-inspiring. In our research of generative concepts, parametric modelling with help of Grasshopper environment proved itself as the best and the most successful intuitive parametrical method to explore design of concepts and to interpret them architecturally. From pedagogic point of view, the main raison why we chose Grasshopper as a tool for development of the box packing algorithm is a potentiality to look at concept as a set of sophisticated relationships between cube elements and to map those relationships graphically and programmatically into a system that allows them interactively play with alternatives. Furthermore, algorithm method and logical thinking for transposing this mathematical model into parametric system can be translated and interpreted to the other different geometrical problems.

In Grasshopper, parameters are objects that represent data, components are objects that do actions like move, orientate, decompose etc. Programs called definitions are created by dragging and dropping components onto a canvas. The outputs to these components are then connected to the inputs of subsequent components. Parametric relationships within the objects are mapped on these connections between components. For transposing special coordinates of corners for all twenty seven elements (Table 1), we use Grasshopper component titled Box2Pt which just create a box defined by two opposite corner points A and B. Of course, it is supposed that the start of packing is $(x, y, z) = (0, 0, 0)$ and this point is corner A for the first packed element at the same time.

Based on the above conception, the definition in Grasshopper can be developed as follows. Component panels provide all elements we need for our form and canvas is the work place where we put our components and set up the algorithm. The values a, b and c are input parameters which will be varied, so we put three numeric slider on the workplace at the start. Instead of assigning fixed values, now parameters that define packing elements are stored as slider variables. By changing values on sliders different proportions of element can be effortlessly created. To realize condition (Eq. 2), values on sliders have to be sorted using components Sort List (Figure, 12).

Using component List Item, corresponding numeric values are transferred to the new objects which save arranged input variables. That allows user to change values of parameters a, b and c without thinking about satisfying conditional relation (Eq. 2).



Figure, 12 - Realization of Eq.2 in Grasshopper definition

As mentioned, a Box2Pt geometry component is needed to create each cubical element, but instead of dragging it 27 times into the canvas, all is needed is just one stated component. That's because the list and data management is a very significant segment of Grasshopper qualities because it's available to have list of data inside just a single parameter or component. Namely, in our case, if A and B input parameters of Box2Pt component contain more than one value, box creating action will be repeated for each pair of corresponding values in the lists. Precisely, if A and B input hold lists of points $\{A_1, A_2, ..., A_{27}\}$ and $\{B_1, B_2, ..., B_{27}\}$, respectively, than there are 27 created boxes as output at once with set of their corresponding corners $\{A_i B_i \mid i = \overline{1,27}\}$.

It's helpful to think of Grasshopper in terms of flow, since the graphical interface is designed to have information flow into and out of specific types of components. Consequently, these two lists of corner points have to be generated first. They can be carried out using geometry component PointXYZ which creates a point using three input parameter named X, Y, Z - its coordinates. For more effective and elegant solution, the task can be reduced on matching six lists $MN, M \in \{A, B\}, N \in \{X, Y, Z\}$. Each of them is a list of 27 numerical values all specified decomposing data from Table 1. Table 2 represents reorganisation of date from Table 1 which is used for optimized realisation in Grasshopper definition (Figure, 13).

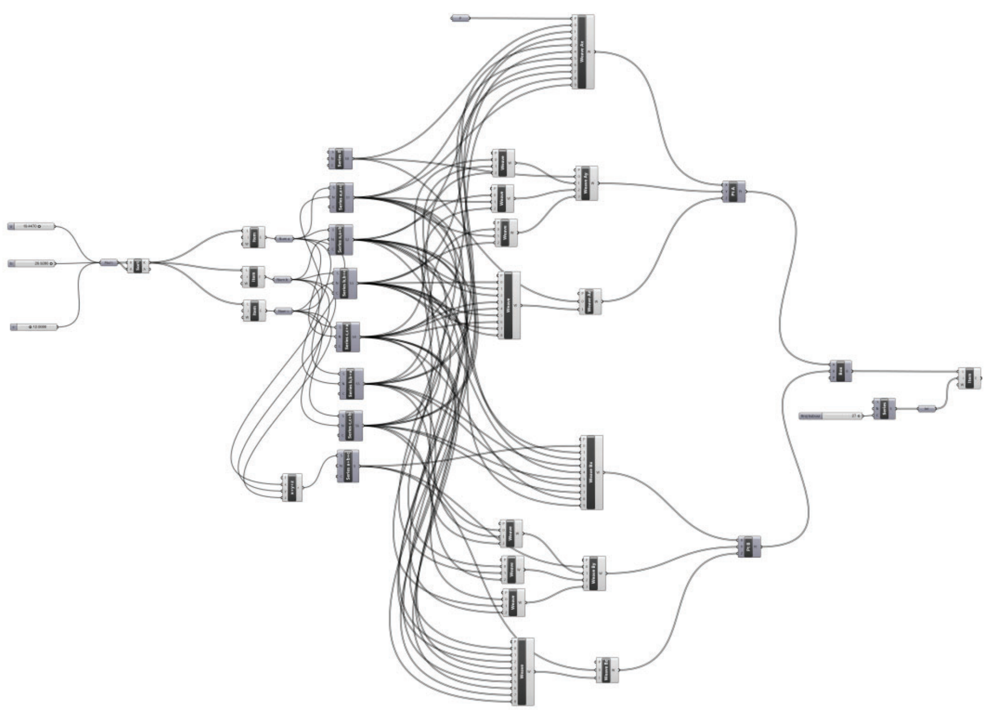
Table 2: Control of management for six numerical lists cubic element's corner

	AX	*AY*	*AZ*		*BX*	*BY*	*BZ*
1	*0*	*0*	*0*		*a*	*c*	*b*
2	*a*	*0*	*0*		*a+c*	*b*	*a*
3	*a+c*	*0*	*0*		*a+b+c*	*c*	*a*
4	*0*	*c*	*0*		*a*	*b+c*	*c*
5	*a*	*b*	*0*		*a+b*	*a+b*	*c*
6	*a+b*	*c*	*0*		*a+b+c*	*a+c*	*b*
7	*0*	*b+c*	*0*		*b*	*a+b+c*	*c*
8	*b*	*a+b*	*0*		*a+b*	*a+b+c*	*b*
9	*a+b*	*a+c*	*0*		*a+b+c*	*a+b+c*	*a*
10	*0*	*0*	*b*		*b*	*a*	*b+c*
11	*b*	*0*	*a*		*b+c*	*a*	*a+b*
12	*b+c*	*0*	*a*		*a+b+c*	*c*	*a+b*
13	*0*	*a*	*c*		*c*	*a+b*	*a+c*
14	*c*	*a*	*c*		*b+c*	*a+c*	*a+c*
15	*b+c*	*c*	*b*		*a+b+c*	*b+c*	*b+c*
16	*0*	*a+b*	*c*		*b*	*a+b+c*	*a+c*
17	*b*	*a+c*	*b*		*a+b*	*a+b+c*	*b+c*
18	*a+b*	*b+c*	*a*		*a+b+c*	*a+b+c*	*a+b*
19	*0*	*0*	*b+c*		*c*	*b*	*a+b+c*
20	*c*	*0*	*a+b*		*b+c*	*a*	*a+b+c*
21	*b+c*	*0*	*a+b*		*a+b+c*	*b*	*a+b+c*
22	*0*	*b*	*a+c*		*c*	*a+b*	*a+b+c*
23	*c*	*a*	*a+c*		*a+c*	*a+c*	*a+b+c*
24	*a+c*	*b*	*b+c*		*a+b+c*	*b+c*	*a+b+c*
25	*0*	*a+b*	*a+c*		*a*	*a+b+c*	*a+b+c*
26	*a*	*a+c*	*b+c*		*a+c*	*a+b+c*	*a+b+c*
27	*a+c*	*b+c*	*a+b*		*a+b+c*	*a+b+c*	*a+b+c*

Figure, 13 - Grasshopper's segment for six numerical lists cubic element's corner management

Weaving of our six lists can be made with six Grasshopper Weave components, one for each of them, but our idea was optimization in sense of minimizing the number of input lists and exploiting numerous repeating of elements and its dependence in resulting output lists. Pedagogically, realization of this concept enables us to demonstrate significance of logic connection within numbers for Grasshopper code optimization, effectiveness and facility for later modifications and extensions. Every Wave component controls the order of a list. P input of this component is the weaving pattern and this determines the order in which our values are arranged, concerning waved. Input Manager of Wave allows to user waving as many lists as needed. Finally, after making all necessary connections within components, all 54 corner points are generated and that automatically produce 27 packed elements (Figure, 14).

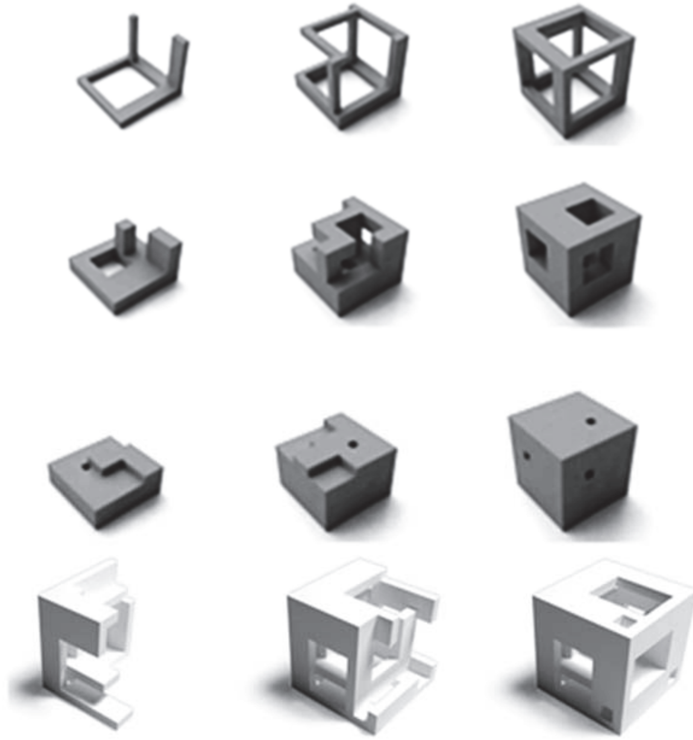
Within the theoretical framework the creation of a form can be understood as a process of individualization where every particular property of a distinctive element is a result of accumulation and interaction of influences, conditions and restrictions. Grasshopper is instantly interactive when we change an input parameter. Every time when we change a, b and c on their sliders, the generated cubical body automatically changes own geometry and adapts to the new variables. Every detail on the resulting geometry is completely and instantly interacting. As shown on Figure, **14**, definition is stated with a new number slider parameter which defines number of packed visible element (from 1 to 27).



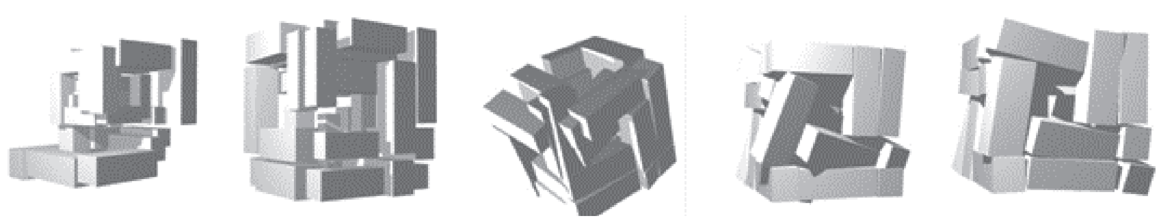
Figure, 14 - Initial Grasshopper definition of Box Packing concept

Described transformation allows us manoeuvring in the development and generation of the resulting packed structure which is not possible when using standard 3D modelling tools. That was the first step in development of our definition. Some of resulting models are shown on Figure, 15.

After initial definition exploration, definition is extended to the distribution of packed form along any specific curve or surface forming an urban complex (multiplication) - related system of functional objects with modification possibility (translation, rotation) for specific cubical element of any individual member (Figure, 16).



Figure, 15 - Rhino models of Grasshopper definition



Figure, 16 – Modification of initial model

5. ADVANCED DEVELOPMENTS

During many years of applying the Box Packing concept in teaching and in organized workshops, computer models were always accompanied by materialization. A Box Packing object can be made practically always from 27 arbitrary, mutually congruent boxes of any dimension. However, if there are any additional requirements, we need a mathematical tool for extracting suitable versions.

5. 1.Implementation of Genetics Algorithms

Further improvements of our system are connected with solving inverse problem – finding specific set of parameters which fulfil defined conditions (minimal or maximal volumetric occupancy etc.). Problem stated on this way (search for solutions) efficiently can be solved using genetic algorithms (GA) [7]. There is no rigorous definition, but it can be said that most methods called GA have at least the following elements in common:

population of chromosomes, selection according to fitness, crossover to produce new offspring, and random mutation of new offspring. Our and most common application is optimization, where the goal is to find set of parameter values that accomplishes given condition, in terms of GA minimize fitness function.

With a defined dimension of the cube, the system becomes a two-parametric. Parameters are two dimensions of congruent elements, and the third dimension is calculated. By defining the appropriate fitness function within optimization using GA [7], we can determine the values of these parameters, thus extracting suitable solutions. The following task is an illustration of this principle. For the given packed cube dimension of 120 $(a+b+c)$, find the dimensions of the cubical elements that form a Box Packing object, so that they make a smaller or larger part of the cube's total volume. Results obtained for several selected values λ, which represents the ratio between the total volume of all form and the volume of the packed elements, are shown in Table 3, and the resulting forms are shown in Figure, 18.

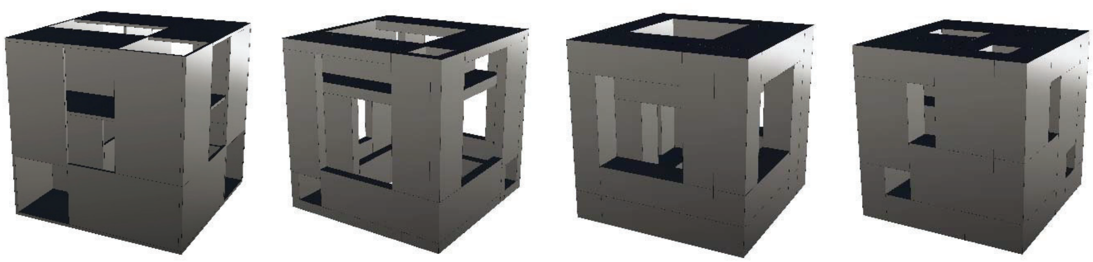
Table 3: Results of the described application of genetic algorithms

Volumetric occupancy (λ)	*0.10*	*0.25*	*0.50*	*0.75*
a	*2*	*7*	*20*	*20*
b	*76*	*27*	*20*	*40*
c	*42*	*86*	*80*	*60*

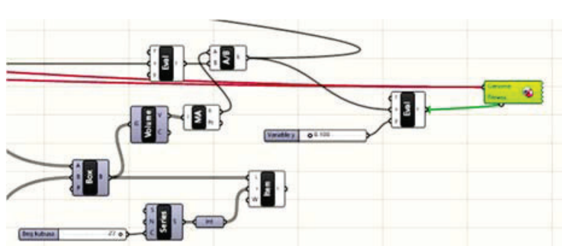
Grasshopper offers powerful component titled Galapagos for implementing GA with given population and fitness function (Figure, 18). Thus implemented genetic algorithm method yields several suitable variations. For presentation in Table 3 and Figure, 17, we selected just one of resulting optimal options. The desired volume ratio is $\frac{V_{Hole}}{V_{Cube}} = \lambda$ which was achieved by finding the minimum of the fitness function

$$\left| \frac{V_{BoxPacking}}{V_{cube}} - \lambda \right|$$

which is within the limits of the permitted error, and equals zero. The population belongs to the following range of integer parametric values $0 < a < 30$, $10 < b < 80$ which means that in the case of the selected cube side of 120, the third dimension is calculated in the following range $10 < c < 110$.



Figure, 17 - Examples of Box Packing objects obtained through the application of genetic algorithms



Figure, 18 - Realization of genetic algorithms method using Galapagos component

5. 2.Box-Counting Dimension

Box-counting fractal dimension of a fractal object is calculated in relation to a raster at different scales [2], [3]. For different side lengths r with $N(r)$, we defined the smallest number of cubical elements with the side length r necessary to cover the shape. In the case of a one-dimensional line segment, two-dimensional unit square and a three-dimensional unit cube the relation between r and $N(r)$ is easy to establish. $N(r)=\frac{1}{r}$, $N(r)=\left(\frac{1}{r}\right)^2$, $N(r)=\left(\frac{1}{r}\right)^3$, in that order.

For more complex geometric shapes, the functional relationship between r and $N(r)$ is not explicit. We will start from the approximate power law relation

$$N(r)=k(\frac{1}{r})^d \quad \textbf{(Eq.3)}$$

which can also be written in the following form

$$Log(N(r))=dLog(\frac{1}{r})+Log(k) \quad \textbf{(Eq.4)}$$

where *d* is the box-counting dimension. In order to calculate *d*, this equality can be observed as a linear equation in the Cartesian coordinate system with coordinate axes $Log(\frac{1}{r})$ and $Log(N(r))$, where the box-counting dimension *d* is the slope of the said line. Through line approximation, using the method of least squares, which has the smallest mean squared deviation from points $Log(\frac{1}{r})$ and $Log(N(r))$, we will calculate $d=d_b$ [3]. The box-counting dimension d_b is often approximated in practice with the slope of the line through a pair of points [10]:

$$d_b \approx \frac{Log(N(r_2))-Log(N(r_1))}{Log(\frac{1}{r_2})-Log(\frac{1}{r_1})} \quad \textbf{(Eq.5)}$$

On the other hand, if we divide the left and right side of the last equation by $Log(\frac{1}{r})$ and switch to the limit when $r \to 0$, the box-counting dimension will be

$$d_b = \lim_{r\to 0}\frac{Log(N(r))}{Log(\frac{1}{r})} \quad \textbf{(Eq.6)}$$

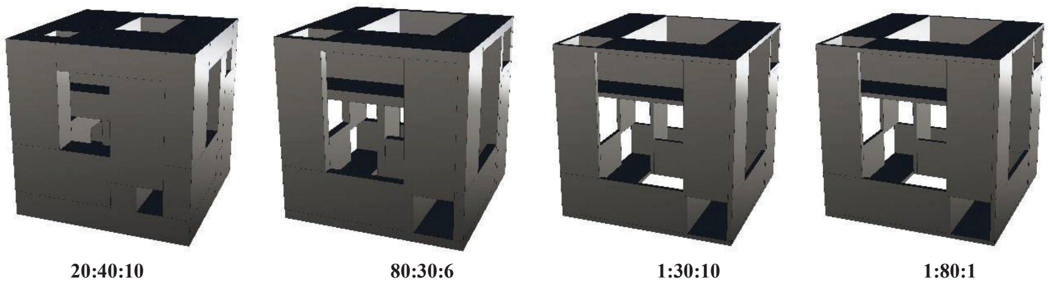
so, we can say that [3]:

$$d_b \approx \frac{Log(N(r))}{Log(\frac{1}{r})} \quad \textbf{(Eq.7)}$$

for a sufficiently small r.

This paper analyzes the box-counting fractal dimension of spatial forms obtained with the use of the Box Packing concept. Grasshopper definition of Box Packing has a built-in additional part which determines the minimum number of boxes $N(r_1)$ and $N(r_2)$ that cover the shape for two selected side lengths r_1 and r_2, calculates the box-counting dimension for each of those values $d_b=d_b(r_1)$ and $d_b=d_b(r_2)$ (Eq.7) and calculates the box-counting dimension $d_b=d_b(r_1,r_2)$ as the slope of the line segment (Eq.5). To extract boxes of the Cartesian grid that cover the shape, we used the Grasshopper component ColissionOneMany.

Selected examples of different ratios between box side lengths $a:b:c$ from the Box Packing concept are shown in Figure, 19. The results are shown in Table 4.



Figure, 19: Examples of Box Packing objects for calculating the box-counting fractal dimension

Table 4: Preview of the box-counting dimension results obtained on the selected examples of Box Packing

	20:40:10	*80:30:6*	*1:30:10*	*1:80:1*
$\boldsymbol{d_b(r_1)}$	*2.92*	*2.70*	*2.71*	*2.06*
$\boldsymbol{d_b(r_2)}$	*2.89*	*2.72*	*2.57*	*1.74*
$\boldsymbol{d_b(r_1, r_2)}$	*2.83*	*2.74*	*2.29*	*1.10*

Box-counting fractal dimension shows the degree to which a geometric shape enters space, in other words, the degree of its compactness or fragmentation in that space. In this sense, we got the results we expected. A visually higher compactness of space is followed by higher values of the fractal dimension. Complete compactness, with a side length ratio of 1:1:1, would correspond to the fractal dimension 3, which matches the Euclid's dimension. Box Packing with a visually linear structure has the lowest fractal dimension.

In addition, by applying genetic algorithms, we were able to select a version with a predefined fractal dimension, which goes beyond the scope of this paper. An open issue is the time required for computer processing. Calculating the fractal dimension in the definition using Grasshopper components is a long process in its own right, and repeating it for different versions within the genetic algorithm would only greatly increase realization time. The problem of process duration optimization remains an open issue of generating new definition using script components.

During the summation and systematization of the results obtained over many years of applying the Box Packing concept and writing this paper, we initially came to the idea of generating fractals that would be covered in detail by some future research. The idea is to generate a fractal with an iterative function system, whose generator [3], [4] will be a Box Packing object. Two examples of the generator and the first iteration of the fractal object are shown in Figure, 20.

Figure, 20: Box Packing as the generator and the first iteration of the fractal object

6. CONCLUSION

Box Packing is a case of mathematical concepts with a significant potential to be exploited in various contexts, be it educational, promotional, artistic or any other related to geometry driven design. Applied in architectural education it is primarily aimed at demonstrating how multidisciplinary, in this case integration of mathematics and geometry enriches design process. In this study we focused on pedagogic aspects of its use, ranging from mastering basic modelling techniques, various simulations, materialization in small scale models and real life objects. Introducing a complex algorithmic approach into the process of building parametric systems has been particularly highlighted and analyzed. It opens a realm of sophisticated programming techniques to architectural students, with a special attention on the combinatorial part. Finally, understanding more advanced techniques like genetic algorithms and abstract notions like fractal dimension, lead to another level of form finding approaches represented in fractalized initial form, promising some further and unexpected design results.

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