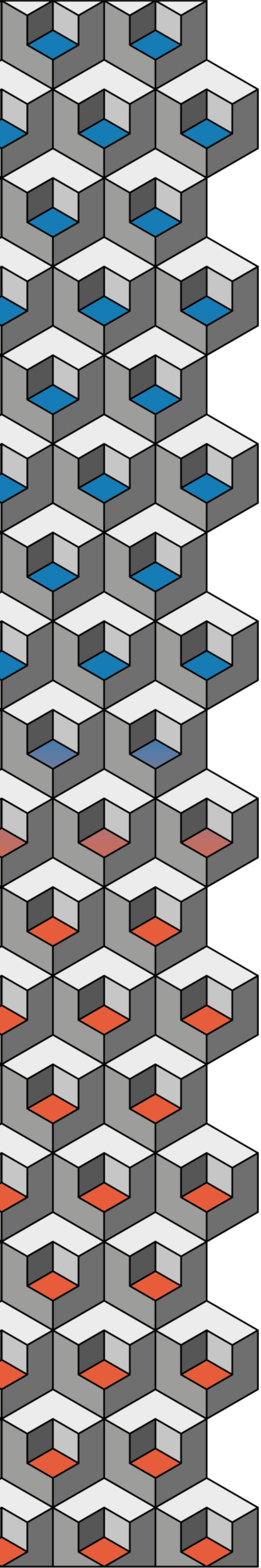

GEOMETRY, GRAPHICS AND DESIGN IN THE DIGITAL AGE

The 9th International Scientific Conference on
Geometry and Graphics

MONGEOMETRIJA 2023

Editors
Ivana Bajšanski
Marko Jovanović

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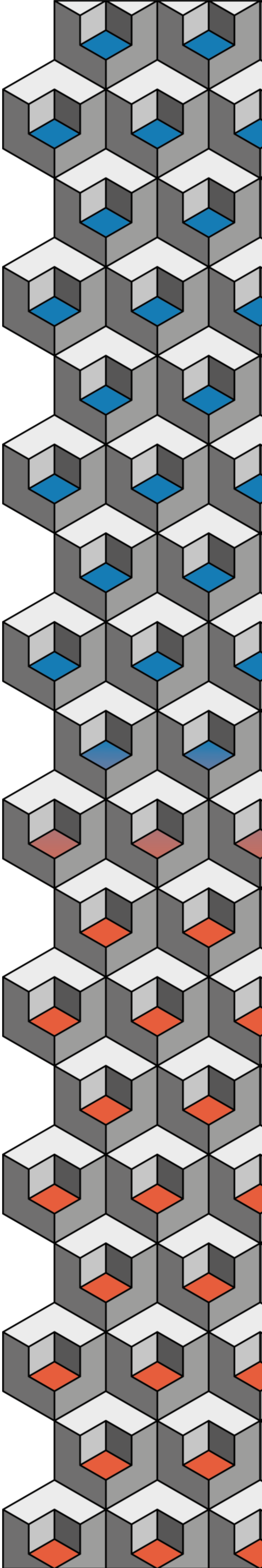
Novi Sad, May 2023

Ivana Bajšanski and Marko Jovanović

MONG2023 Conference chairs



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ENCODING/DECODING CAPITALS OF CLASSICAL ARCHITECTURAL ORDERS BY USING FRACTAL GEOMETRY: ESTABLISHING METHODOLOGY

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Abstract

In most cases, artefacts are differentiated in terms of style they belong to – mainly visually, not mathematically. So, the main research questions of this study are both how to numerically encode stylistic regularities (peculiarities) as geometric indicators of artefacts morphology and how to decode them, namely to identify architectural style those artefacts belong to. Columns, namely their capitals are chosen as the most distinctive elements among artefacts. To elaborate on the validity of the defined principles of the aim-related methodology, a few representatives (capital samples) from each of three fundamental classical architectural orders (Doric, Ionic, and Corinthian) are used.

The subject of this Paper is to establish relevant indicators of capital qualification, capital classification, and thus, referred architectural order identification. The verification of those indicators is performed by processing two sets of capitals contours (that belong to the mutually equidistant transverse and equiangular radial section planes) of each of the selected samples (namely digital 3D models). The narrower research aim is to point out that it is possible to encode not only chosen but also any other capital – by using the mentioned indicators of fractal and non-fractal nature (as a control one). The wider research aim refers to a possibility to identify order a concrete fragment of capital belongs to in terms of recognising it computationally (as confidently as possible from the mathematical probability point of view) based on the established research methodology principles.

Finally, it is possible to conclude that changes of the analysed indicators trendlines behaviour (expressed by changes of its slope, roughness, continuation, etc.) accurately/precisely describe morphology-wise variations of a form that could point out subject-related stylistic variation, as well. So, this Paper demonstrates not only the fact that architectural orders capitals are true fractal objects, but rather how fractal analysis as a tool can be used to scientifically numerically encode/decode their certain characteristics (fractal features) of single- or multi-scale nature.

Keywords: fractal dimension, stylistic encoding/decoding, capital qualification, capital classification, architectural order identification

1 INTRODUCTION

Fractal geometry is broadly implemented in the domains of architecture and urbanism. For example, a (multi)fractal approach is used to define systemic planning strategies [1], to describe the design of modern structures [2], to characterise and to predict crack growth in (reinforced) concrete – either by the usage of machine learning [3] or without it [4], to classify both static and dynamic textures [5], etc. However, the fact that there is a limited number of papers directly related to this research topic, turns out to be the main reason to broadly scientifically investigate the related domain of interest.

In the field of architecture, a majority of studies differentiate artefacts in terms of style they belong to – mainly visually [6], [7], [8]. From the mathematical point of view, Carl Bovill performs fractal analysis in terms of measuring/estimating the fractal dimension based on so-called box-counting method [9], [10]. Capo notes that if artefacts are interpreted fractally, it could be said that the general form is not what counts the most, but rather, the way in which parts hold together [11]. But, although the architectural orders fractal nature has already been investigated (as previously mentioned), capital qualification and classification, including order identification methodology, criteria, and numeric indicators definition have not been provided in this way yet. Therefore, the main research questions of this study are both how to numerically encode stylistic regularities (peculiarities) as geometric indicators of artefacts morphology and how to decode them, namely to identify architectural style those artefacts belong to.

Consequently, the central hypothesis refers to the statement that it is possible to describe those stylistic regularities (peculiarities) by using fractal geometry, primarily. But, why would fractal geometry be used in this investigation at all? Precisely, because it is assumed that: (a) capital contour lines which belong to section planes of various slicing directions are of mono/multifractal type (given that their morphological characteristics correspond to the principles of generating natural fractals), and (b) change of their level of detail ("roughness") by changing section planes, is an indicator of complexity "spreading" (changing) across the capital mass.

The subject of this Paper is to establish morphometric/geometric indicators in the form of descriptors and qualifiers, enabling so: to encode, namely to qualify capitals, to classify them and, consequently, to decode them, namely to identify/recognise classical architectural orders stylistically. With respect to such an idea, it becomes possible to estimate probability levels of: (a) intra-similarity of capitals assumed to belong to the same order, and (b) inter-dissimilarity of those assumed to belong to different ones.

The subordinate hypothesis refers to the statement that intra-order behaviours of descriptors of fractal nature (that change as a consequence of the contours complexity changing and expressed in a form of trendlines) are to be consistent so that they can be substituted by "averaged" behaviours.

The narrower research aim is to point out that it is possible to encode not only chosen, but also any other capital – by using the established indicators of fractal and non-fractal nature (as a control one), according to the developed methodology. The wider research aim refers to a possibility to identify order a concrete fragment of capital belongs to in terms of recognising it computationally (as confidently as possible from the mathematical probability point of view).

2 METHODOLOGY

In order to reach the goals tasked, digital 3D models of real capitals are used. To define the mentioned capital qualification, capital classification and order identification methodology-criteria as well as related indicators, two vector-wise contour line sets (hereinafter: "contours") of each concrete digital 3D model are generated [12], then exported into a raster format (.jpg) and, consequently, processed by a fractal analysis software. Contours from the first set belong to the mutually equidistant transverse section planes of digital 3D models, while those from the second set belong to their mutually equiangular radial ones.

As mentioned in Section 1, those indicators are expressed by both several descriptors of fractal/non-fractal nature (as a control one) and several corresponding qualifiers which are represented by arithmetically averaged values of the analysed descriptors – expressing so their global "from-section-to-section" changing rule. The nature of the established descriptors and qualifiers will be broadly explained in Sub-subsection 2.3.2.

2.1 Starting Considerations

To draw scientifically acceptable conclusions, two groups of prerequisites and constraints have to be satisfied: Morphometry/Geometry-related and ImageAnalysis-related.

2.1.1 Morphometry/Geometry-related Prerequisites and Constraints

For results to be scientifically mutually comparable, several morphometry/geometry-related prerequisites and constraints ought to be met: (a) chosen capitals originals must conform to known canonical proportions (including sort and level of detail) [6], [7], [8], (b) those originals have to be selected so as to be of the same material, having dimensions not too uneven mutually[1] (due to the fact that the level of capital detail corresponds to its dimensions/measures and applied building material), (c) digital 3D models ought to represent originals as accurately as possible, and (d) their "digital

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[1] In this way, a distance measured between two consecutive dots on each of the analysed capitals "contours in reality" would be represented by one pixel on its corresponding raster image and, consequently, each computed (multi)fractal dimension would not be significantly affected by that "scale" [9].

masses" must be unified – represented in the form of Closed Breps/Meshes (so as corresponding contours to remain closed, as well).

2.1.2 ImageAnalysis-related Prerequisites and Constraints

Several prerequisites and constraints must be taken into account in case of using (raster) ImageAnalysis (multi)fractal plugins. Namely, raster images (of previously created transverse and radial contours) ought to be prepared so that: (a) consisting contours be single pixel wide, (b) their overall sizes (in pixels) be as larger as possible (because (multi)fractal dimensions of contours in such "larger" images tend to be closer to theoretical values), and (c) be created in or converted into a binary mode (Black&White) [13].

2.2 Equipment and Software

The following equipment and software are used: Laptop Computer Dell Vostro 3580 15.6" FHD Intel Core i7 (8th Gen) 8565U 8GB Radeon 320, Robert McNeal and Associates Rhinoceros 7.0 with Grasshopper plugin (to generate vector contours and calculate relevant descriptors), Adobe Illustrator CC 2015 (to prepare previously generated vector contours and export them as raster images), ImageJ v1.53t with FracLac plugin (to analyse raster images and compute (multi)fractal dimension), and Microsoft Office Excel 16 (to calculate qualifiers and create relevant charts).

2.3 Methodology Setup

2.3.1 Representative Samples Selection

To elaborate on the sustainability of established capital qualification, capital classification, and order identification criteria as well as the validity of defined principles of the aim-related methodology, a few representatives of each of three fundamental (basic) architectural orders (Doric, Ionic, and Corinthian) are used. Columns are chosen as the most distinctive elements among artefacts, precisely their capitals, because of unique/order-specific design of those column parts [6], [7], [8]. From the research point of view (its aim), Greek/Roman orders sub-differentiation is declared not important.

Despite the fact that a larger number of examples provides more reliable conclusions, according to the nature of this research (establishing methodology, exclusively), only nine capital samples are chosen – three of each order (Fig. 1.). Capital samples selected according to Sub-subsection 2.1.1 are appropriate representatives from the theoretical point of view (having typical/canonical configuration without any "excessive" ornamentation).

Fig. 1. Nine Analysed Representative Samples of Three Fundamental Architectural Orders: Triptych of Triplets of Selected Doric, Ionic, and Corinthian Capital Samples (from left to right)

The zones of column masses shown in Fig. 2. are taken into account. According to literature, for Doric and Corinthian orders, the mentioned zone is a part of the column between astragal (positioned on top of the column shaft) and architrave (epistyle), while for Ionic one, that zone includes astragal [6], [7], [8].

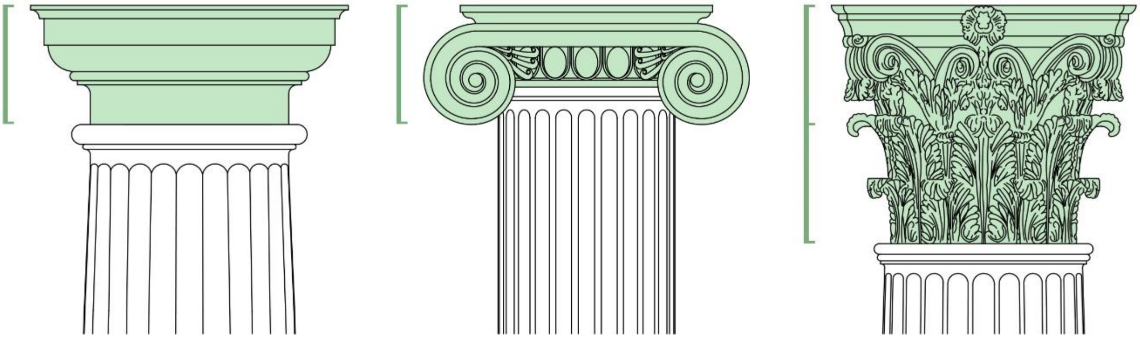

Fig. 2. Marked Column Zones that Define Capitals: Doric, Ionic, Corinthian (from left to right)

It is obvious that the higher level of morphometric/geometric similarity between chosen capitals originals and 3D models as their digital representations is, the more reliable level of accuracy of the obtained results (and thus, the sustainability of the proposed methodology) will be. In spite of the mentioned fact, according to the aims of this research, capitals digital 3D models are acquired (retrieved from the relevant Internet sources)[2] so as to satisfy prerequisites and constraints listed in Sub-subsection 2.1.1 as much as possible (being aware that some of them are not initially digitised and, hence, might be slightly interpreted in a "freestyle" authorial way).

Sizes of selected samples (digital 3D models of capitals) are mutually uniformed by scaling lower diameters of related shafts (to get the value of 2 theoretical modules (hereinafter: "M")). Thus, capitals heights also become proportionally/canonically mutually complied ($H_{Doric} : H_{Ionic} : H_{Corinthian} = 1M : 1M : 2M$, respectively), despite the general fact that the upper shaft diameter (dimensionally related to the diameter of the lower capital base) diminishes proportionally as the shaft height increases [6], [7], [8], [14]. This "adjustment" provides the most sustainable cross-referencing between intra- and inter-order indicators (descriptors and qualifiers, respectively) that refer to the corresponding transverse and radial contours.

2.3.2 Indicators Definition

According to the purpose of this research, two types of descriptors are defined: a primary one – represented by (multi)fractal dimension, and a control one of non-fractal nature – represented by Euclidean area. Control descriptor is foreseen because of the following facts: (a) contours of geometrically mutually different planar shapes may have the same (multi)fractal dimension, and (b) (multi)fractal dimensions of all transverse contours of Doric capital are identical in general (theoretically equal to 1.0000). Two subtypes of each of these descriptors are also provided, according to the transverse and radial slicing. So, four descriptors are introduced: FractalDescriptorTransverse/Radial (hereinafter: "FDT"/"FDR"), and AreaDescriptorTransverse/Radial (hereinafter: "ADT"/"ADR").

FDT (namely FDR) is a descriptor related to the distribution of (multi)fractal dimensions of contours located in each of transverse section planes (namely radial ones) – expressed by a unitless value. ADT (namely ADR) is a descriptor related to the distribution of areas bounded by the previously generated corresponding sets of contours – expressed by square metres.

To scientifically estimate probability levels of intra-similarity of capitals assumed to belong to the same order and inter-dissimilarity of those assumed to belong to different ones, two types of qualifiers are defined, as well: the first one which relates to arithmetically averaged (multi)fractal dimensions, and the second one which relates to arithmetically averaged areas – both obtained separately for each triplet of contours defined by section planes of the same ordinal number, namely of the same slicing position in the corresponding triplet of capital samples of the same order. Each of these types is also sorted into subtypes according to the applied – "transverse"/"radial" slicing directions, as follows: FractalMeanTransverse/Radial (hereinafter: "FQT"/"FQR"), and AreaMeanTransverse/Radial (hereinafter: "AQT"/"AQR"). Having in mind the previously mentioned descriptors typology as well as the fact

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[2] https://archibaseplanet.com/, https://www.turbosquid.com/, https://www.cgtrader.com/, https://sketchfab.com/feed, https://3dwarehouse.sketchup.com/, https://www.pond5.com/3d-models/ [accessed: 2022-12-10].

that qualifiers are derived from them, it is obvious that the firstly listed qualifier of both subtypes is also the primary one, while the other one, including its subtypes, is the control.

3 INDICATORS OBTAINING

3.1 Sets of Transverse/Radial Contours Generating

Transverse slicing is carried out by the usage of a set of equidistant section planes perpendicular to the axial capitals axes and distributed along the whole of its height (Fig. 3. (left)). With regard to the radial slicing, due to specific symmetry characteristics of Ionic and Corinthian orders, for all analysed capital samples (including Doric), the mentioned slicing is performed by using a set of mutually equiangular section planes positioned through their axial axes so as to cover symmetry zones of capital masses (2x1/4=1/2 of the mass) shown in Fig. 3. (right).

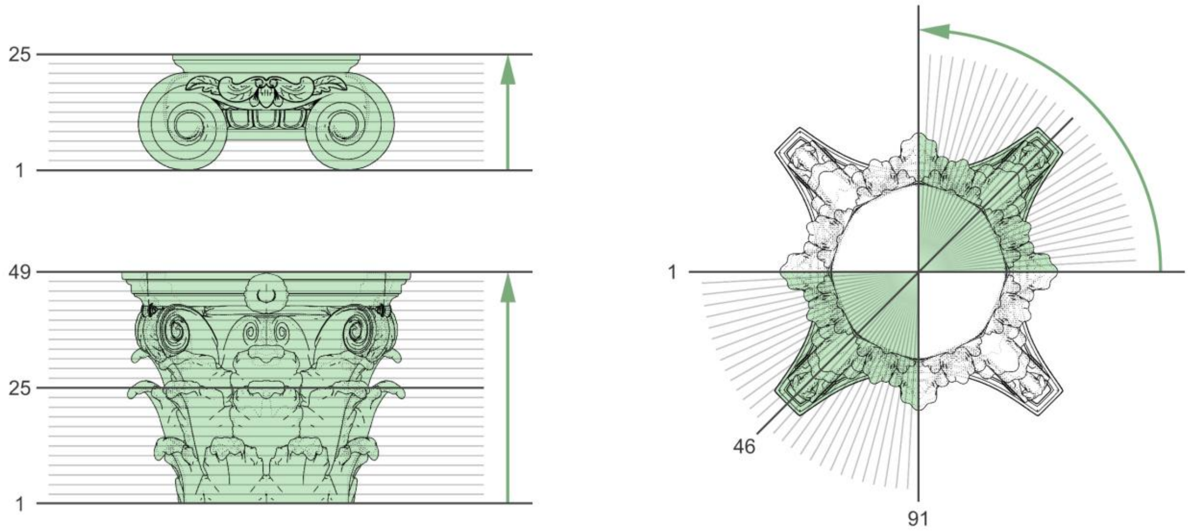

Fig. 3. Slicing Zones of Capital Samples: Set of Transverse Equidistant Section Planes (left), and Set of Radial Equiangular Section Planes (right)

To achieve a higher level of accuracy of the subject-related estimation, namely of the mentioned descriptors obtaining (and of qualifiers, consequently) one centimetre (1.0cm) stands for the linear step (used for transverse slicing) and one degree (1.0°) stands for the angular step (used for radial slicing). In spite of the fact that the smaller step is, the better results will be, the adopted values of steps can be declared satisfied from the research-aim point of view (establishing methodology, exclusively).

Because of a large number of section planes, corresponding contours generation is automatically performed by the usage of a specially designed GrasshopperAlgorithm (definition), illustrated in Fig. 4. (top). For this research point of view, values of computed fractal dimensions that correspond to each concrete capital contour, are not important in themselves, but rather their mutual comparability is essential. Accordingly, a fractal dimension estimation approach that implies a fixed sample boundary (“constant study area”, not “variable” one) is used – depicting more accurately a fractal dimension change as section planes change [15].

So, transverse/radial slicing is performed by using a unique cutting box/cylinder which is slightly larger[3] than the bounding box/cylinder of the largest analysed capital sample, ensuring thus that (multi)fractal dimension is not significantly affected by “scale”, mentioned in Sub-subsection 2.1.1 (Fig. 4. (bottom-left/bottom-right)). Due to the usage of fixed (constant) sampling boundary and as a result of inevitably variable morphometry of the capitals in adopted slicing directions, there will be a significant metric difference between the aforementioned dimensionally invariant image boundary

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[3] In that case, cutting edges as image borders do not touch each of generated contours and, hence, they are not included in (multi)fractal dimension computation.

(defined by cutting box/cylinder) and the actual contours in it. Therefore, the expected (multi)fractal dimension values will range between 0 and 2 [15].

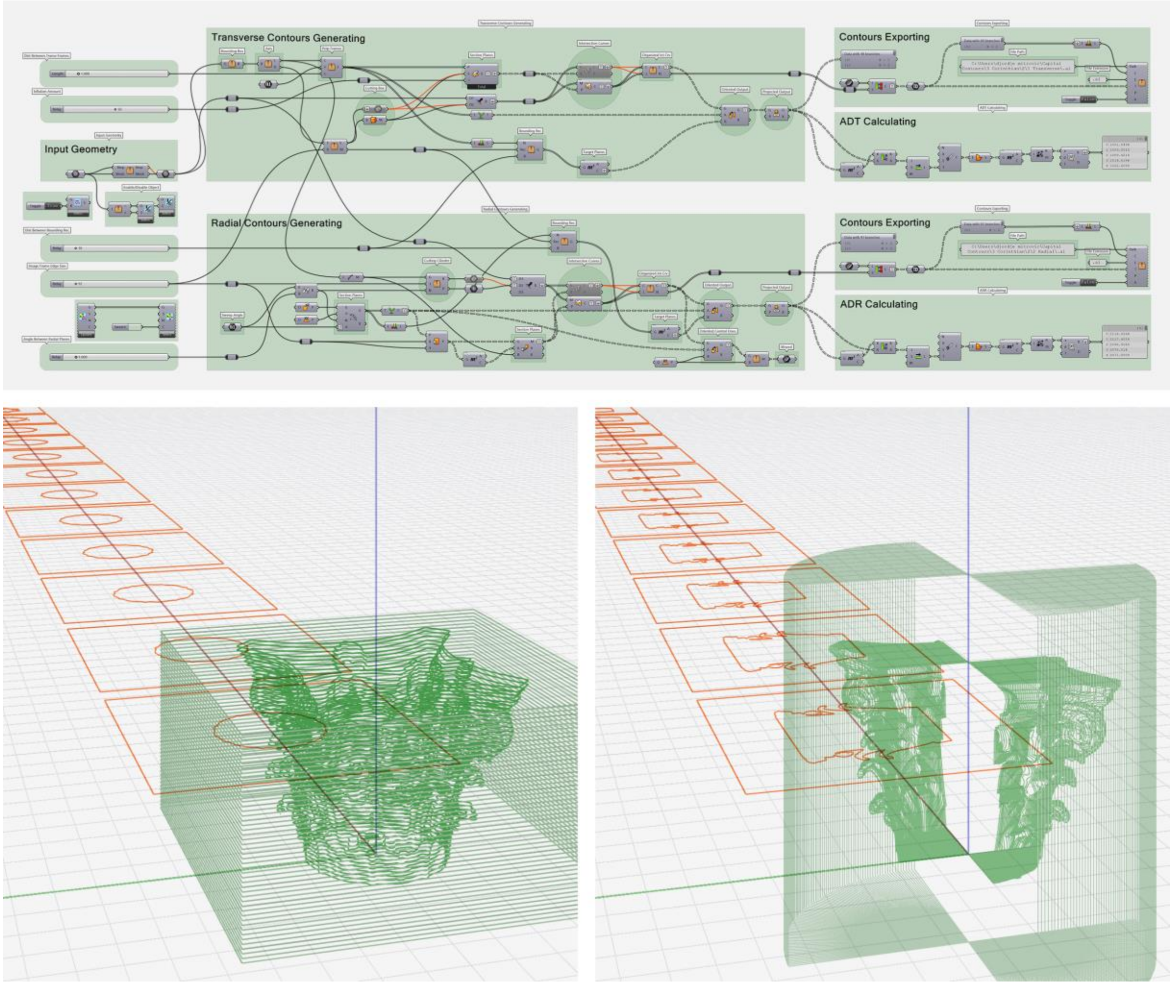

Fig. 4. Illustration of the Created GrasshopperAlgorithm and Its Interactive Manifestation in Model Space: Definition Used to Generate Capitals Contours by Their Automated Slicing (top), Sequence of Transverse Slicing (bottom-left), and Sequence of Radial one (bottom-right)

3.2 Raster Images Preparing

To obtain a scientifically sustainable comparison of intra- and inter-order capitals characteristics (represented by investigated indicators), according to Sub-subsection 2.1.2, image size in pixels of all raster contour representations ought to be the same. To achieve efficient image processing from a temporal point of view (including RAM consumption) being enabled to preserve the level of complexity of contours, image size (in pixels) is empirically set to 5315 x 5315 (having 150 ppi) that is assumed as optimal in this case. All raster images are exported as “maximum quality” JPEGs (RGBs) and, then, converted into binary ones (Black&White).

3.3 Descriptors Acquiring

3.3.1 FDT and FDR Computing

Software Calibration and Setup. To achieve intermediate and final outputs as accurately/precisely as possible, the FracLac plugin is previously calibrated. So, binary raster images of several fractal entities of known (theoretical) fractal dimension are used (Sierpinski Triangle – generated by Iteration Function System, as well as Koch Curve and Minkowski Sausage – generated by Lindenmeyer System) (Fig. 5.).

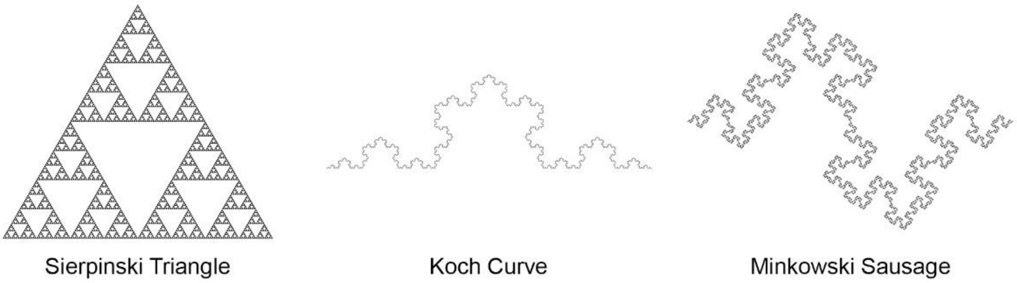

Fig. 5. Monofractals Used for FracLac Calibration and Setup

Those images are prepared in the same way as the experimental ones (respecting the facts underlined in Subsection 3.2). For “sustainable” calibration, the “kth-iterations” of the mentioned commonly known fractal entities are empirically adopted as “adequate”, because their levels of detail correspond to that of the contour (among all generated) which is declared/estimated as the most detailed one. In this way, values of fractal dimension of those “kth-iterations” of calibration entities are obtained. To prove the validity of the software setup, raster images of Circle and Square as “fractal initiators” of the lowest level of detail (having a theoretical fractal dimension of 1.0000) are also implemented. However, regardless of such calibration guided by the morphometry/geometry-related prerequisites and constraints underlined in Sub-subsection 2.1.1, according to the FracLac manual [13], the estimated deviation of computed fractal dimension values from theoretical ones generally is in a range of 1–5%. According to the mentioned, computed fractal dimension of circle/square can not be 1.0000, but slightly different and used as such in the following calculations (Table 1.).

Table. 1. Fractal Dimensions of Used Monofractals and Initiators: Theoretical Values, Computed Values, Corresponding Correlation Coefficients (r^2), and Deviation Rates

Monofractals and Initiators Names	**Theoretical Values**	**Computed Values**	**Correlation Coefficients (r^2)**	**Deviation Rates***
Sierpinski Triangle	1.5849	1.6113	0.9993	1.6657%
Koch Curve	1.2618	1.3134	0.9977	4.0894%
Minkowski Sausage	1.5000	1.4966	0.9978	0.2267%
Circle	1.0000	1.0315	0.9953	3.1500%
Square	1.0000	1.0137	0.9957	1.3700%

*Expressed According to Theoretical Value in the Function of Acceptable Software Accuracy (1–5%)

For (multi)fractal dimension computation, box-counting method (hereinafter: “BC”) is used, because it is confirmed as an effective and appropriate method for fractal dimension estimation (hereinafter: “D_B”) [9], [10]. According to the adopted ImageSize (in pixels), following BC options are taken by default: (a) twelve grids are set (without *“rotate”* and *“random”* options checked) and (b) regarding scaling method represented by *“Default Sampling Sizes”,* linear series of 100 different box sizes are chosen (box sizes increase by a fixed increment of 4 px over a range from the min. size of 5 px to the max. size of 45% of the image size). Only *“check pix”,* as a *“Special Scan Option”,* is checked manually to do a pixel check before scanning, namely to count only boxes that contain within a certain ratio of foreground to background pixels (in order to minimize edge effects). To do so, a min. density value is set to 0.20

(default value)[4], while a max. density value is set to 1.0 (default value, too)[5]. As all of the mentioned presets are in common for the overall D_Bs computation, they are saved to a file and used for the following automation.

Process Automation. Owing to a chosen number of capital samples and, consequently, to a large total number (1116) of generated contours (transverse of (297)[6] and radial of (819)[7]). D_B computation is automated by the usage of internal FracLac "batching". So, nine resulting batch files for transverse slicing (for 3 capital samples (hereinafter: "1st", "2nd", and "3rd") of each of 3 orders) and nine resulting batch files for radial slicing (for the same entry) are inserted into Excel as .txt files (for the following numerical processing, namely for qualifiers calculation and for descriptors/qualifiers trendlines generation in the form of charts).

Achieved Accuracy and Precision of Computed D_Bs. The validity of the obtained D_Bs is proven by a statistical evaluation of the distribution of the analysed datasets (automatically by FracLac itself), namely by the computed Correlation Coefficients r^2 (that express determination for each regression line – showing the relationship between the logarithm of foreground pixel count and box size obtained for each concrete D_B calculation) [13]. As those values of r^2 (related to each separate D_B) are close to 1.0, the correlation between data analysed can be declared highly statistically correlated and acceptable as such (the overall range of the achieved values of r^2 in this research is from 0.7978 to 0.9993).

Remark. In case of having slices consisting of more mutually separated (but closed) contours, computed (multi)fractal dimension D_B is an average value of all "partial" ones in the analysed slice.

3.3.2 ADT and ADR Calculating

Due to the mentioned large number of generated contours, according to the ADT/ADR descriptor definition (stated in Sub-subsection 2.3.2), calculation of its values is also automatically performed by the usage of a specially designed GrasshopperAlgorithm (definition).

Remark. In case of having slices consisting of more mutually separated (but closed) contours, the resulting area is the sum of all "partial" ones (when one contour is nested into the other, the resulting area is subtracted one).

3.4 Qualifiers Acquiring

3.4.1 FQT and FQR Calculating

According to the FQT/FQR qualifier definition (stated in Sub-subsection 2.3.2), based on previously computed corresponding FDT/FDR descriptor, its calculation[8] and chart creation are performed by the usage of Excel.

3.4.2 AQT and AQR Calculating

According to the AQT/AQR qualifier definition (stated in Sub-subsection 2.3.2), based on previously computed corresponding ADT/ADR descriptor, its calculation[9] and chart creation are performed by the usage of Excel, also.

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[4] Software counts only samples with more than 20% of the overall pixel density.

[5] Software counts only samples with the total density less than 100% of pixels.

[6] There are generated 25 transverse contours (section planes) for each of 3 analysed Doric/Ionic capital samples, and 49 transverse contours (section planes) for each of 3 Corinthian ones ((25x3)+(25x3)+(49x3)=297).

[7] There are generated 91 radial contours (section planes) for each of 3 analysed Doric/Ionic/Corinthian capital samples ((91x3)+(91x3)+(91x3)=819).

[8] $FQT_{(i)}$ = (1st $FDT_{(i)}$ + 2nd $FDT_{(i)}$ + 3rd $FDT_{(i)}$) / 3, i=(1, n); $FQR_{(i)}$ = (1st $FDR_{(i)}$ + 2nd $FDR_{(i)}$ + 3rd $FDR_{(i)}$) / 3, i=(1, n), where "n" is the total number of transverse/radial slices.

[9] $AQT_{(i)}$ = (1st $ADT_{(i)}$ + 2nd $ADT_{(i)}$ + 3rd $ADT_{(i)}$) / 3, i=(1, n); $AQR_{(i)}$ = (1st $ADR_{(i)}$ + 2nd $ADR_{(i)}$ + 3rd $ADR_{(i)}$) / 3, i=(1, n), where "n" is the total number of transverse/radial slices.

4 RESULTS

To elaborate the results comprehensively and to cross-reference them in a scientifically acceptable way, two types of numerical research outputs are defined, namely: intermediate and final. Intermediate outputs tell of intra-order similarity (stylistic peculiarities) from the morphology point of view, while the final ones express, from the same point of view, both inter-order dissimilarity and acceptable tolerance of a presumed intra-order similarity expectedness (as criteria that ought to be used to classify capitals conceptually, namely to perform order identification properly).

4.1 Intermediate Outputs

Intermediate outputs are graphic representations of the obtained numerical values of descriptors of fractal and non-fractal nature, describing so intra-order similarities. They are in the form of six charts (3x2=6). Charts 1(a/b), Charts 2(a/b), and Charts 3(a/b) show transverse-wise/radial-wise trendlines triplets of descriptors of both defined types (fractal and non-fractal), related to the triplet of capital samples of the same order (hereinafter: “1st FDT”/“1st FDR”, “2nd FDT”/“2nd FDR”, “3rd FDT”/“3rd FDR” and “1st ADT”/“1st ADR”, “2nd ADT”/“2nd ADR”, “3rd ADT”/“3rd ADR”).

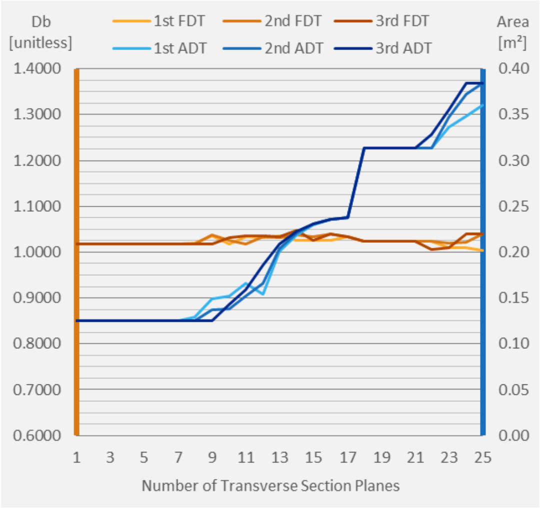

Chart 1a. Intra-order “Doric” Similarity: Transverse-wise Descriptors Trendlines related to the Triplet of Corresponding Capital Samples

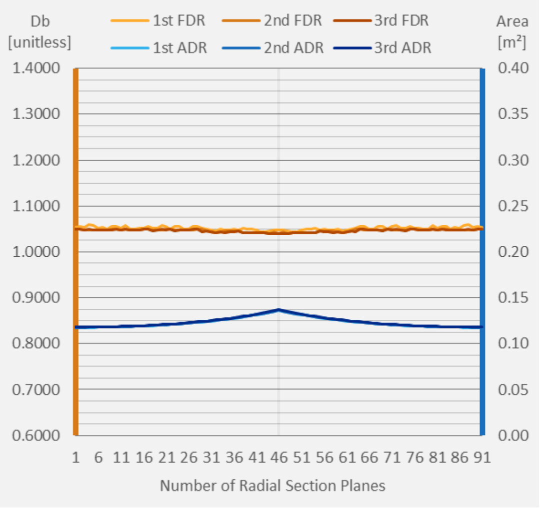

Chart 1b. Intra-order “Doric” Similarity: Radial-wise Descriptors Trendlines related to the Triplet of Corresponding Capital Samples

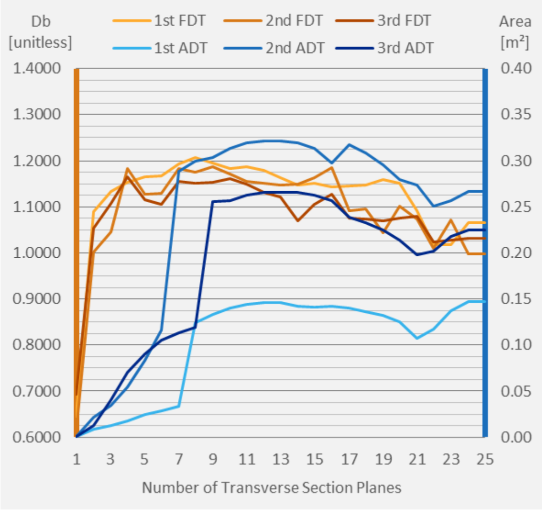

Chart 2a. Intra-order “Ionic” Similarity: Transverse-wise Descriptors Trendlines related to the Triplet of Corresponding Capital Samples

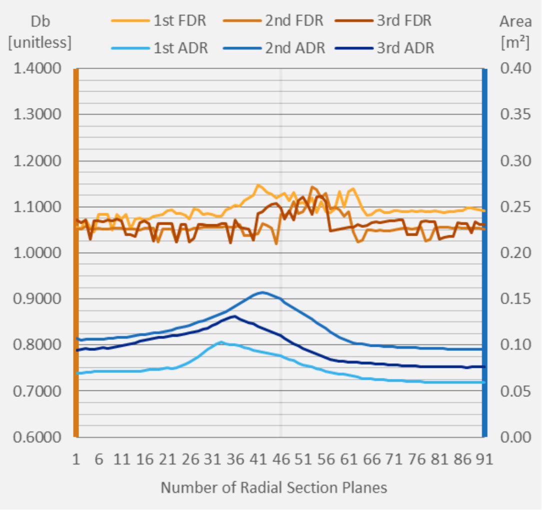

Chart 2b. Intra-order “Ionic” Similarity: Radial-wise Descriptors Trendlines related to the Triplet of Corresponding Capital Samples

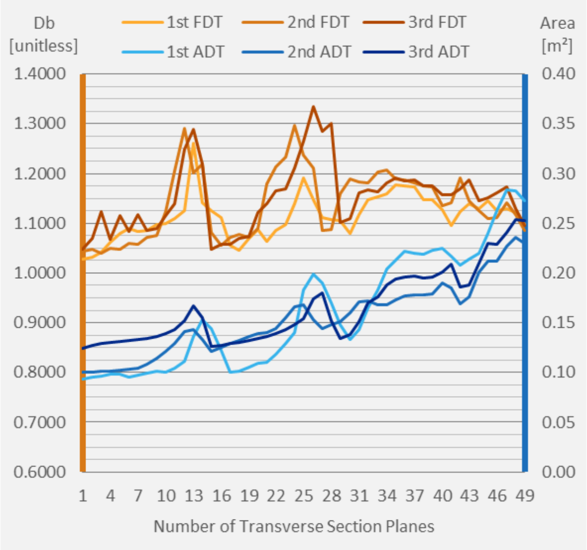

Chart 3a. Intra-order "Corinthian" Similarity: Transverse-wise Descriptors Trendlines related to the Triplet of Corresponding Capital Samples

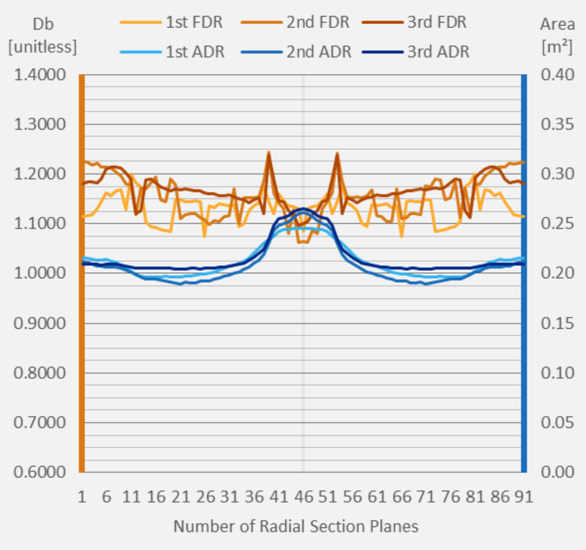

Chart 3b. Intra-order "Corinthian" Similarity: Radial-wise Descriptors Trendlines related to the Triplet of Corresponding Capital Samples

4.2 Final Outputs

Two groups of final outputs are defined. The first group refers to graphic representations of the obtained numerical values of qualifiers (derived from the corresponding descriptors of fractal and non-fractal nature), describing so inter-order dissimilarities. The second group relates to graphic representations of the obtained numerical values of those qualifiers thresholded by minimum and maximum values of the same type that correspond to the section planes with an identical ordinal number (namely transverse/radial slicing position).

The qualifiers from the first group are presented in the form of four following charts (2x2=4). Charts 4(a/b) show transverse-wise/radial-wise trendlines triplets of qualifiers of fractal nature, while Charts 5(a/b) show transverse-wise/radial-wise trendlines triplets of qualifiers of non-fractal nature. Each qualifier trendline substitutes as mean the corresponding triplet of descriptor trendlines by "averaging" it (hereinafter: "Dor FQT"/"Dor FQR", "Ion FQT"/"Ion FQR", "Cor FQT"/"Cor FQR" and "Dor AQT"/"Dor AQR", "Ion AQT"/"Ion AQR", "Cor AQT"/"Cor AQR").

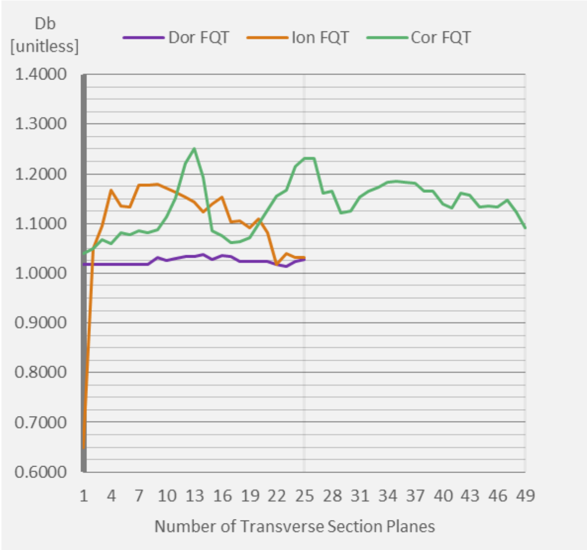

Chart 4a. Inter-order Dissimilarity: Transverse-wise Fractal Qualifiers Trendlines that Substitute Triplets of Corresponding Descriptors Trendlines

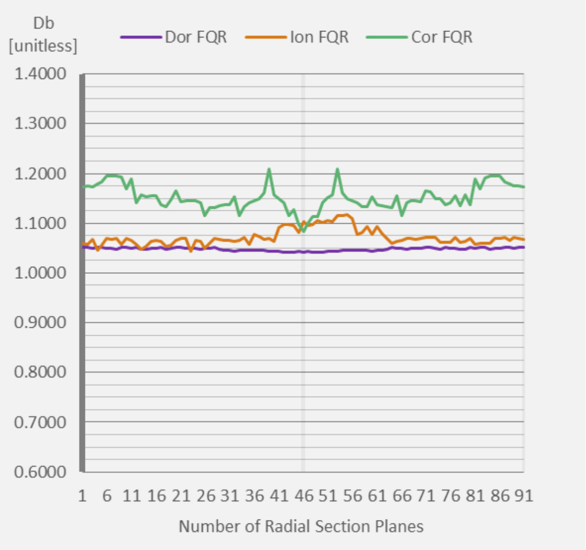

Chart 4b. Inter-order Dissimilarity: Radial-wise Fractal Qualifiers Trendlines that Substitute Triplets of Corresponding Descriptors Trendlines

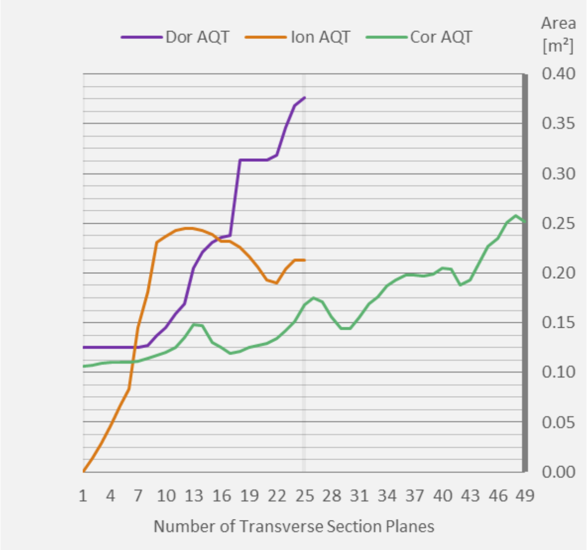

Chart 5a. Inter-order Dissimilarity: Transverse-wise Non-fractal Qualifiers Trendlines that Substitute Triplets of Corresponding Descriptors Trendlines

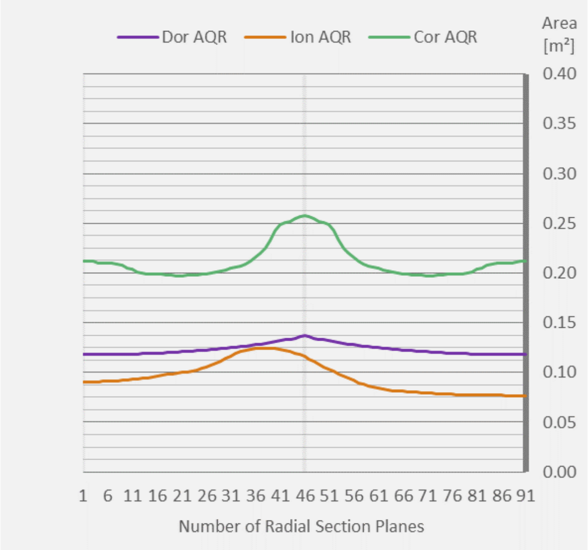

Chart 5b. Inter-order Dissimilarity: Radial-wise Non-fractal Qualifiers Trendlines that Substitute Triplets of Corresponding Descriptors Trendlines

The qualifiers from the mentioned second group are presented below in the form of two charts. Charts 6(a/b) show transverse-wise/radial-wise trendlines of “thresholded” qualifiers of fractal and non-fractal nature that relate to one of the examined orders only – the Corinthian one (because transverse descriptors and qualifiers trendlines of its capitals have the largest x-axis domain ranged from 1 to 49 (namely the largest number of applied section planes)). Each chart consists of two sets of elements. Each set consists of four elements: corresponding qualifier trendline as a mean one (Cor FQT/Cor FQR namely Cor AQT/Cor AQR), minimal threshold trendline (hereinafter: “Min FDT”/“Min FDR” namely “Min ADT”/“Min ADR”), maximal threshold trendline (hereinafter: “Max FDT”/“Max FDR” namely “Max ADT”/“Max ADR”), and marked tolerance zone of the presumed intra-order similarity expectedness (hereinafter: “Tlz FDT”/“Tlz FDR” namely “Tlz ADT”/“Tlz ADR”).

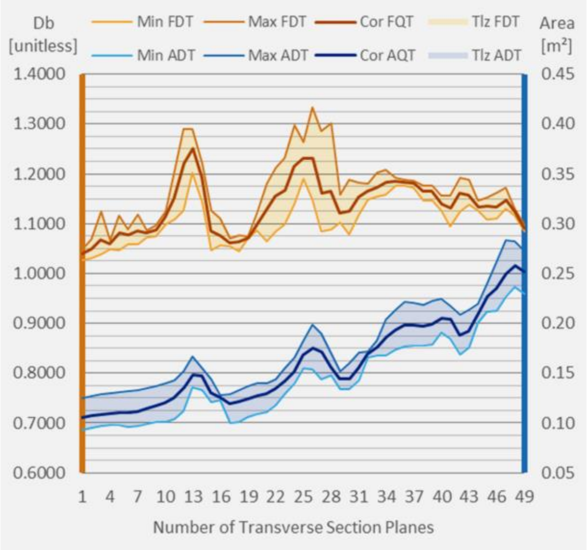

Chart 6a. Corinthian Order Identification/Recognition: Transverse-wise Tolerance Zones outlined by Corresponding Threshold Trendlines

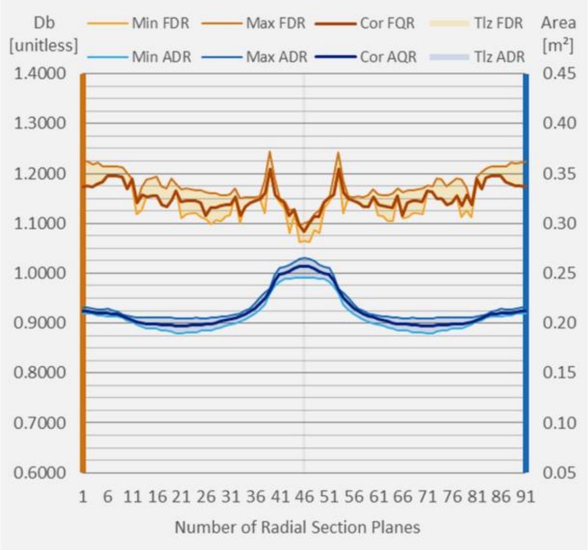

Chart 6b. Corinthian Order Identification/Recognition: Radial-wise Tolerance Zones outlined by Corresponding Threshold Trendlines

5 DISCUSSION

The following analytical overview of the findings obtained in this study covers both intermediate and final outputs. It offers their balanced explication and summarises key considerations as well as research implications, providing general insight into their significance and potential applications. Explication of intermediate outputs relates to the interpretation of the obtained descriptors whose behaviour is graphically represented in Charts 1(a/b), Charts 2(a/b), and Charts 3(a/b), while explication of final outputs relates to the interpretation of the obtained qualifiers whose behaviour is graphically represented in Charts 4(a/b), Charts 5(a/b), and Charts 6(a/b).

5.1 Intermediate Outputs Explication

According to Charts 1(a/b), Charts 2(a/b), and Charts 3(a/b), per each examined order, descriptors trendlines of fractal nature (of their both subtypes – transverse/radial) display similar behaviour pattern within the belonging triplet. This similarity is seen in terms of consistent peak and valley positions, slight amplitude diversity along y-axis, and slight interval diversity along x-axis, namely the slicing direction. The first kind of diversity arises due to the existence of a triplet of contours (defined by section planes of the same ordinal number per corresponding triplet of capital samples) characterised by mutually different levels of complexity. While the lastly listed is a result of the presence of unavoidable morphometric variations among capital samples and their various interpretations, both respecting a concrete canonical rule (in terms of proportional-wise height division) – another type of diversity is caused by the presence of certain interpretations of canonical rules themselves. Certain local trends that relate to the behaviour of trendlines characterised by their more significant ascent or descent (notable in some sets of consecutive section planes) can be declared a consequence of the mentioned more pronounced changes in the geometry (complexity) of corresponding contours. Such a behaviour refers not only to the trendlines of fractal nature but to those of non-fractal one, as well.

But, in contrast to the trendlines of fractal nature, the non-fractal ones are smoother curves generally, mostly having a more gradual flow. From this point of view, the Doric order is an exception, because its trendlines are rougher (having a larger number of corners and rises), particularly transverse-wise ones that are predominantly ascending. Additionally, trendlines of non-fractal nature of their both subtypes (within the same triplet) are insignificantly diverse along both chart axes for Doric and Corinthian orders, whereas for the Ionic one that diversity is more notable. Thus, the behaviour of trendlines of non-fractal nature (regardless of their subtype) within the Ionic triplets for both listed parameters (smoothness/roughness namely amplitude/interval diversity) can be considered a “mixture” of the two previously explained (of Doric and Corinthian).

Unlike transverse slicing, the non-fractal descriptor values, related to the radial one, change more gradually and the diversity of trendlines is less pronounced – significantly smaller within the same triplet of the Ionic order, even smaller in the case of the Corinthian order, and the smallest in the case of the Doric order, because of their corresponding morphometric/geometric characteristics and the canonical rules applied. Additionally, according to this slicing (performed as described earlier in Subsection 3.1), owing to a specific “diagonal” radial symmetry of Doric and Corinthian capitals, the symmetry of their corresponding descriptor trendlines that refers to radial section planes numbered 46 (displaced from their initial position by rotating for 45 degrees) is to be expected, also. However, in the case of the Ionic order, certain deviation from that regularity is notable, because of its specific morphological/canonical configuration. One should not expect an absolute already mentioned "diagonal" radial symmetry due to the presence of building/crafting imperfections on capitals, and thus, their digital 3D models. This is especially pronounced for trendlines of a fractal nature, given that the fractal dimension is computed (estimated iteratively), not calculated exactly/theoretically.

By comparing generally the behaviour of triplets of trendlines of non-fractal nature, it is obvious that such a behaviour can also be considered inter-order consistent (valorised by the usage of the same parameters as in the case of fractal ones), but with a lower peak and valley density along the x-axis, namely the slicing direction. Amplitude and interval diversity (also existent owing to the aforementioned unavoidable morphometric variations among capital samples and their various interpretations) are more pronounced for non-fractal descriptors (because this descriptor type is more prone to “from-section-to-section” changes of its values, given that the area bounded by a contour generally changes more significantly than a contour complexity itself). Additionally, this diversity is much larger for transverse slicing due to more notable changes of descriptor values along the height of the capital. Despite a more pronounced amplitude diversity within the triplet of trendlines of the same order and of non-fractal nature (regardless of their subtype), the behaviour of trendlines of fractal nature can be considered more consistent, particularly in the case of Ionic order.

Finally, the firstly elaborated intra-order similarity of corresponding trendlines of fractal nature, confirms the statement that the intra-order behaviours of fractal descriptors (that change as a consequence of the contours complexity changing and expressed in the form of trendlines), are to be consistent so as to be substituted by “averaged” behaviours, proving so the subordinate hypothesis. Thus, the statement that it is possible to describe subject-related stylistic regularities (peculiarities) by using fractal geometry, primarily – is confirmed too, proving so the central research hypothesis.

5.2 Final Outputs Explication

As mentioned, by proving the subordinate hypothesis it becomes possible to generate qualifiers trendlines expressing so-called “averaged” behaviours. According to Charts 4(a/b) and Charts 5(a/b), a significant inter-order dissimilarity, namely diversity among those “averaged” behaviours is obvious within corresponding triplets of qualifiers trendlines of both fractal and non-fractal nature (regardless of their subtype). According to the transverse slicing, one should note that, unlike the domain of Corinthian qualifiers trendlines, ranged along x-axis from 1 to 49, the domain of Doric/Ionic ones is ranged from 1 to 25, due to a difference in capital heights (as shown in Fig. 2. and Fig. 3.). Therefore, the transverse-wise trendlines are not factually inter-order comparable throughout their flow: Doric and Ionic trendlines are shorter than Corinthian one twice along x-axis (after a mutual uniforming of their sizes by respecting canonical proportions), because heights of the capitals of those two orders are also two-times lower.

The Doric transverse-wise qualifier trendline is very specific given that it is predominantly parallel to the x-axis, as expected, because of the similarity of fractal dimensions of contours within their corresponding sets. Additionally, the averaged values of fractal dimension are close to 1.0000 as a result of geometric characteristics of the mentioned contours of Doric capital samples in that slicing direction (which are squares and circles only). Another extreme trendline behaviour is of the Ionic transverse-wise qualifier: its trendline reaches the averaged fractal values which are smaller than 1.0000. The aforementioned is a result of a gradual increase of the contour size value (in this slicing direction/bottom-up) from an infinitely small one (obtained for a significantly small fulfilment of a fixed sampling frame (boundary) at the initial slicing position).

The observed differences in domains of the qualifier trendlines as well as in distribution of qualifier values themselves (regardless of their subtype) can be declared the main numerical indicators of inter-order differentiation. On the other hand, behaviours displayed by the corresponding trendlines (in terms of peak/valley distribution and ascent/descent steepness as well as amplitude values and peak/valley density) represent the main graphic indicators of the mentioned inter-order differentiation. Unlike transverse slicing, in the case of a radial one (of both fractal and non-fractal nature) less pronounced peak/valley values (and thus, amplitude ones) are present. Such a differentiability (valorised by the usage of the same parameters as in the case of fractal nature) is also obviously expressed by the trendlines of qualifiers of non-fractal nature.

With regard to a capital stylistic peculiarities decoding phase, the elaborated inter-order dissimilarity assessment turns out to be its necessary subphase. Namely, Charts 6(a/b) show that the process of decoding is to be possible within this study only for capitals of the analysed classical architectural orders. Therefore, in order decoding phase to be more effective, a rough capital classification is to be adopted as its first step.

However, to be able to perform that decoding with a higher level of reliability, an acceptable tolerance of the presumed intra-order similarity expectedness is introduced (as a range of the descriptor values that correspond to each of its generated contours). In this study, as an example only, the mentioned tolerance is set for each specific section plane by defining a pair of thresholds represented by the minimal and maximal value of the three descriptors that relate to the concrete analysed triplet of capital samples and correspond to the section plane of the same ordinal number (slicing position). The second step (fine tuning) of decoding implies an assessment of the level of order-belonging (in terms of checking the mentioned first step) – by comparing each value of its descriptor (obtained per each generated contour, namely per each transverse/radial section plane) with the corresponding value of the qualifier for the same section plane taking into account its tolerance range bounded by the corresponding thresholds. Each belonging to that zone declares order identification/recognition successful.

Finally, although the usage of descriptors and qualifiers of control (non-fractal) nature is not necessary to estimate a level of inter-order dissimilarity, namely to classify a capital roughly, they ought to be used to assess a level of intra-order belonging, due to the fact that the capital (or any other artefact) would not have to belong to an estimated/concrete order relying on an exclusive criterion of fractal nature, because a morphologically different form and, hence shapes and sizes of its contours can be of the same fractal features.

6 CONCLUSIONS AND FUTURE RESEARCH

This study demonstrates how stylistic regularities (peculiarities) of artefacts morphology can be numerically encoded/decoded from the methodological point of view – by using fractal geometry, primarily. To do so, relevant indicators of artefact qualification, classification and architectural style identification/recognition are established. Those indicators relate to two sets of artefacts (namely their digital 3D models) contours that belong to the mutually equidistant transverse and equiangular radial section planes. Besides indicators of fractal nature, those of non-fractal nature as control are also introduced. It is observed that changes of indicators trendlines behaviour are capable of accurately/ precisely describing morphology-wise variations of artefacts form (namely their stylistic variation). The investigation is performed by the usage of classical columns, namely their capitals as the most distinctive elements among artefacts.

It is concluded that both transverse and radial sets of contours are proper for stylistic encoding (their qualifying). Contrary, for reliable stylistic decoding (capitals classification and order identification/ recognition) radial slicing is more suitable from the practical point of view than the transverse one, since the number of section planes applicable in that slicing direction is invariant of capitals height. On the other hand, transverse slicing is more suitable for morphology-wise segmentation, owing to the distinctive canonical proportions that stylistically differentiate architectural orders capitals just along their axial axes (namely capitals heights).

The established methodology applied to the architectural orders capitals exclusively, provides a very clear insight into the fact of how fractal analysis can be used from the same viewpoint broadly – in the field of architecture/urbanism, in general. So, the purpose of this paper is not only to demonstrate that architectural orders are true fractal objects, but rather how fractal analysis as a tool can be used to scientifically numerically encode/decode their certain characteristics (fractal features) of single- or multi-scale nature.

Although the classification phase is introduced in this paper to demonstrate only conceptually how it is possible to perform more effective decoding from this research point of view (representing its first step), it is obvious that it can be applied in the form of a separate procedure, as well – when one wants to classify capitals in the context of the analysed classical architectural orders, exclusively. The second step of the decoding phase implies the assessment of a level of order-belonging and it is defined methodologically also. In order to confirm the suggested methodology practically, and to prove its scientific sustainability more reliably, future research in this domain will include a significantly larger set of representative samples (capital digital 3D models) not only per each analysed order, but per Tuscan, Composite, and others.

Accordingly, it will be possible to determine computationally various characteristics of probability distributions of descriptors and qualifiers, such as values of Mean and +/- Standard Deviation (as tolerance limits), that can be considered the already mentioned thresholds with highly acceptable statistical reliability. So, it will be possible to determine more properly whether the analysed capital belongs to a concrete architectural order or not. Another future research task refers to possibilities: to decode the order a concrete fragment of capital belongs to – by recognising it computationally (as confidently as possible from the mathematical probability point of view) and to perform morphology-wise segmentation (as a precondition of semantic one) – by employing artificial intelligence in terms of machine learning, primarily.

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