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THE USE OF WEBER'S FOCAL-DIRECTORIAL PLANE CURVES AS APPROXIMATION OF TOP VIEW CONTOUR CURVES AT ARCHITECTURAL BUILDINGS OBJECTS

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Abstract. *One of the major aims when researching some problems in architectural design of buildings is to fully understand and adequately apply the underlying scientific foundations that architects use in their projects. In this paper we analyze the application possibilities of the Weber's focal-directorial curves in the approximation of ground-base contour line of architectural objects i.e. buildings. Thus, a Weber's curve with m foci and n directrices was defined. Furthermore, particular qualifiers were introduced in order to estimate the level of adequacy of the conducted approximation. The importance of the research can be sought in the fact that the exact procedure has been created with its applicability in architectural-urban design of contemporary forms as well as in the domain of the historical heritage and conservation in the sense of the creating proper geometrical models for further computer aided use*.

Key words: *Weber's curves, parabola, architectural objects, approximation.*

1. INTRODUCTION

The approximation of the contours of architectural objects with various curves is one of the problems that was dealt with by several authors. The importance of solving this problem is reflected for instance in the easier possibility of restoration of historic

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buildings. In that sense, Duvernoy and Rosin concluded that “the purpose of research on design problems in historic architecture is usually to understand and to reveal the hidden theoretical sciences that the architects applied in their projects”, [4]. Some papers however studied the analysis and design of masonry arches, masonry arch bridges and buildings, [3], [7], [9] and [10].

Gonzalez et al. determined a curve, which best fits an architectural arch and its analytical equation, and then they apply this method to the 23 arches in Palau Guell, [8]. Authors have concluded that Gaudi experimented with the four types of conic curves (circle, ellipse, parabola, hyperbola) and the two types of hyperbolic-cosine curves (Rankine, catenary). Ginovart et al. determined the form of Taragona Amphitheatre. They analyzed the following geometrical forms: ellipsis and ovum, [6].

Plane Erdös-Mordell’s curves have been of particular interest in our previous researches [1], [15]. Thus, the Erdös-Mordell inequality of a triangle, defined by a relationship between three-focal sum and three-directrix sum, has been investigated.

Curves, generated as a locus of points with the constant sum of distances from two or more foci have been so far widely investigated, not only by mathematicians but by artists, architects and engineers as well. The application of multifocal plane curves in the approximation of top-view as well as of various side-views contours of diverse buildings has been researched in [21]. Thus, the use of ellipse and egg curve has been analyzed in [17] and [5]. On the other hand, it has been found out that three focal curves turn out to be of a great importance in determining optimal geometric properties of infrastructural corridors as well as in solving in some loci and optimization problems, [18]. In order to achieve the above mentioned goals, curves being a locus of points with a constant sum of distances from multi foci and directrices, have been generated and derived in [20]. In the research [22] the comparison of conics and hyperbolic-cosine curves types in Gaudi’s geometric forms approximation has been carried out. The importance of such curve approximation can be found as a base for further computer modelling for the sake of preservation of the cultural heritage [14].

The visualization of Weber’s curves and surfaces with at the most three foci and/or three directrices as well as the investigation in particular problems of optimization has been discussed in the following papers [2], [19]. In recent years it has become possible to enlarge the family of surfaces suitable for application in architecture by constructing new surfaces, [23].

What makes this paper distinctive? In this paper a new mathematical form of Weber’s curves of an arbitrary number of foci and directrices has been developed. Furthermore, a procedural methodology for the approximation of contours of architectural forms with two planes of symmetry using Weber’s curves has been created. The quality of the Weber’s curves approximation has been estimated using both the coefficient of determination and the absolute deviation of the real contour from the approximation curve. Likewise, the quality of approximation has been estimated through the comparison between the Weber’s curve herewith defined and the parabola (which can be also treated as a Weber’s curve but with a single focus and a single directrix). The practical use of the proposed method has been carried out on two different architectural structures, i.e. buildings.

2. METHODOLOGY

In the course of approximation base points of the ground-base contour curve are selected on objects that are approximated (points at the same height level). These points are the common points of the approximation curve's segments as well.

The approximation procedure demands the selection of the so called base points of the ground-base contour points of the analyzed object which are to be approximated (points at the same height level). These points are the fixed points of the approximation curve's segments. Taking into account the double-axial symmetry of the conic like ground-base of the object, three out of four ending points on the axes of symmetry are chosen as the base points.

In order to achieve as good as possible approximation of top view contour of an existing architectural-urban structures by a morphologically adequate Weber's curve, it is necessary to generalize the relationship established in Erdös-Mordell's curve 3 foci and 3 directrices [15], defining a locus of points in a plane for *m* points and *n* lines. Accordingly, the following definition is introduced in [20].

A Weber's focal-directorial curve is a locus of points in a plane of a constant sum of scaled distances from *m* fixed points (foci) and *n* fixed lines (directrices):

$$W_{fd}^{[\alpha_1,\dots,\alpha_m][\beta_1,\dots,\beta_n]}(S):\alpha_1 R_1+\dots+\alpha_m R_m+\beta_1 r_1+\dots+\beta_n r_n=S,\ \ m,n\geq 1 \tag{1}$$

where $R_1,\dots, R_m$ and $r_1,\dots, r_n$ are Euclidean distances of the point $T(x, y)$ from the foci $F_1,\dots, F_m$, and from the directrices $d_1,\dots, d_n$, respectively, and the scale factors $\alpha_1,\dots, \alpha_m$, $\beta_1,\dots, \beta_n \in$ R are the Weber's weight coefficients (at least one is of a non-zero value) and $S=const.$

The smallest value of the parameter $S=S_0$ for which locus (1) is a nonempty set represents Fermat-Weber's set of points $F=W_{fd}^{[\alpha_1,\dots,\alpha_m][\beta_1,\dots,\beta_n]}(S_0)$. When all the Weber weight coefficients equal to 1, all the previously defined curves are of a harmonic proportion such as at Paladio, since according to their genesis they can be related to an arithmetic mean of distances of all points from foci and directrices [20].

2.1. Approximation I

As known a parabola is the genealogically prime curve of the discussed Weber's curves, and it is a locus of points with the same distance from the focus and the directrix, (see Fig. 1). A point *T* that satisfies this condition and can be expressed by the following equation:

$$W_{fd}^{[1][-e]}(0):R_1-e\cdot r_1=0,\ \ e=1 \tag{2}$$

where R_1 and r_1 are the Euclidean distances from the point $T\,(x, y)$ to the focus $F\,(0, Y_F)$ and the directrix d_1: $y = Y_d = Y_F + p$, $p>0$, respectively. The parameter $e = 1$ represents the eccentricity of the parabola. The parabola treated as a Weber's focal-directorial curve given in (2) can now be expressed as follows

$$\sqrt{x^2+(y-Y_F)^2}=\left|y-Y_d\right|. \tag{3}$$

Because of the ground base contour's double-axial symmetry the choice of boundary conditions for the parabola of the equation in explicit form:

$$\langle parab \rangle : y = ax^2 + bx + c, \tag{4}$$

implies the defining of the following triple of points

$$\begin{aligned} &B(0, Y_B) \in \langle parab \rangle \Rightarrow c = Y_B, \\ &A(X_A, 0)\ ; \overline{A}(-X_A, 0) \in \langle parab \rangle \Rightarrow a = -\frac{Y_B}{X_A{}^2},\ b = 0. \end{aligned} \tag{5}$$

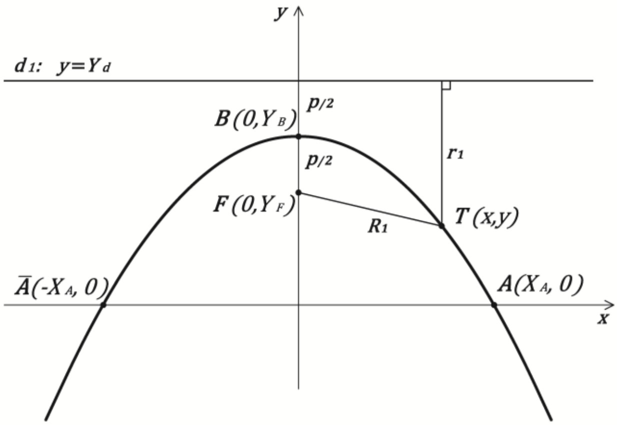

Fig. 1 Genesis of a Parabola (Weber's curve with a single focus and a single directrix)

The Weber's curve with a single focus and a single directrix given by the equation (2) where $R_1 = r_1$ transforms into its implicit form (3). After squaring the equation (3) and substituting the boundary conditions (5) we get:

$$y = -\frac{1}{2p} x^2 + Y_B \tag{6}$$

where $p = X_A^2 / 2Y_B$; $Y_F = Y_B - p/2$ and $Y_d = Y_B + p/2$.

2.2. Approximation II

The Approximation II involves the Weber's curve of two foci and three directrices (see Fig. 2), defined as follows:

$$W_{fd}^{[1,1][1,1,k]}(S) : R_1 + R_2 + r_1 + r_2 + kr_3 = S, \tag{7}$$

where $R_1 = \sqrt{x^2 + (y-f)^2}; R_2 = \sqrt{x^2 + (y+f)^2}; r_1 = |y-d|; r_2 = |y+d|; r_3 = |y|$ and $k, S \in$ R, of the boundary points A, B and C:

$$\begin{aligned} &A(X_A,0)\in W_{fd}^{[1,1][1,1,k]}(S)\Rightarrow 2\sqrt{X_A{}^2+f^2}+2d=S,\\ &B(0,Y_B)\in W_{fd}^{[1,1][1,1,k]}(S)\Rightarrow 2d+(k+2)\ Y_B=S,\\ &C(X_C,Y_C)\in W_{fd}^{[1,1][1,1,k]}(S)\Rightarrow \sqrt{X_C{}^2+(Y_C-f)^2}+\sqrt{X_C{}^2+(Y_C+f)^2}+2d+kY_C=S. \end{aligned} \tag{8}$$

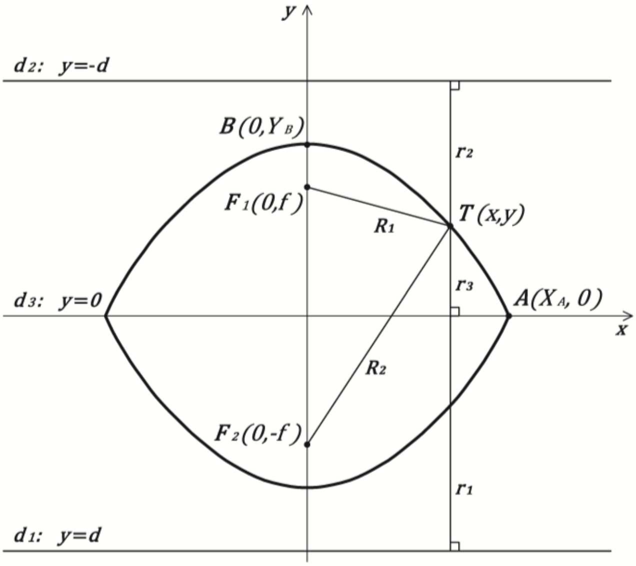

Fig. 2 Genesis of a Weber's curve with two foci and three directrices

2.3. The approximation procedure

Since this paper provides only basic principles of the approximation procedure, the analysis of the achieved procedural precision is based on the obtained values of two following qualifiers:

1) coefficient of determination R^2, [17] and
2) Δy - *fitting error*.

The large value of the calculated coefficient of determination is not necessarily the indicator of the maximal matching obtained by the approximation and therefore a new qualifier (Δy - *fitting error*) is being introduced. It represents the maximal difference of the specific pair of corresponding points' ordinates (of the same abscissa). Each pair of corresponding points consists of one point of a discrete set ***P*** and of one point of the Weber's curve i.e. of the parabola being the obtained approximation.

3. Results

In this paper the proposed methodology of the approximation is carried out on two geometric models taken from architectural practice. In order to satisfy the initial definitions (analysis of plane curves) the case study demands two architectural structures whose facade sheets are vertical (i.e. cylindrical surfaces) with the basement edges on the same height level.

The first object is the "Dorton Arena–Paraboleum" – USA, Deitrick W.H. and Nowicki M., 1952., [11]. Dorton Arena features parabolic design that wisely combined architecture and engineering. The arena received the First Honor Award of the American Institute of Architects in year 1953. In 2010 an initiative was launched to have the arena designated as a UNESCO World Heritage Site.

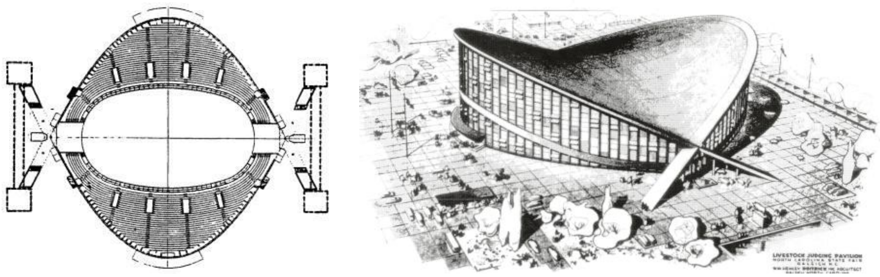

Fig. 3 The Paraboleum–Dorton Arena
(Source: http://www.remhaus.pl/pol_pl_staw_areny.htm, Accessed: 2015-3-25;
Source: http://www.ncstatefair.org/facilities/dortonhistory.htm, Accessed: 2015-3-25)

Taking into account that the contour of its horizontal basement consists of two axially symmetric parabola segments this building is set as the object in order to determine the level of precision of the conducted approximation (see Fig. 3). The dimensions of the ground base are taken from the technical drawings given at [16].

The ground-base contour curve of the control model is defined by the discrete set of points ***P*** containing 200 manually selected points P (X_P, Y_P). The points are positioned in a local Descartes coordinate system (axes x and y are chosen so as to coincide with Arena ground-base's axes of symmetry). In order to increase the points' selection precision out of the graphical representation a vectorization of the control model's raster drawing is previously carried out (drawing from the Fig. 3 – left).

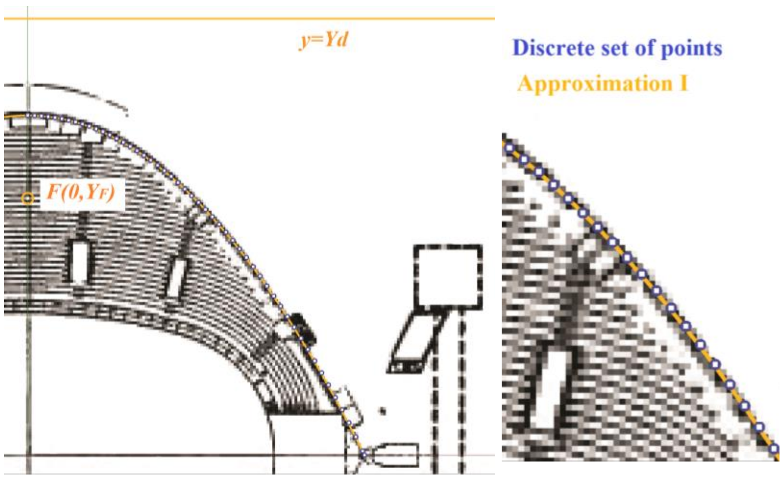

Fig. 4 The contour of the Dorton Arena's floor plan approximated by parabola (Weber's curve with a single focus and a single directrix)

For the same reason, and in accordance with the fact that a floor plan has a double axial symmetry, the collection of points is reduced to a quarter of the contour. Hence, boundary conditions X_A and Y_B are defined as follows

$$\begin{aligned} X_A &= max\,(x), \quad \text{for all } (x, y) \in \boldsymbol{P}, \\ Y_B &= max\,(y), \quad \text{for all } (x, y) \in \boldsymbol{P} \end{aligned} \tag{9}$$

and their numerical values (given in metres) are $X_A = Y_B = 45.72\ m$.

For the selected set of points $\boldsymbol{P}$ (see Fig. 4), the numerical value of the coefficient of determination is $R^2 = 0.99995$, while the numerical value of the fitting error is $\Delta y = 0.123\ m$. The values for both analyzed quantifiers applied to the Dorton Arena's floor plan indicate the great precision of the conducted approximation procedure.

The second building that is to be analyzed is The Great Hall of the Textile Fair (in Serbian: Velika hala Sajma tekstila) in city Leskovac – Srbija, Balgač E. and Cvetić M., 1959., [12], [13].

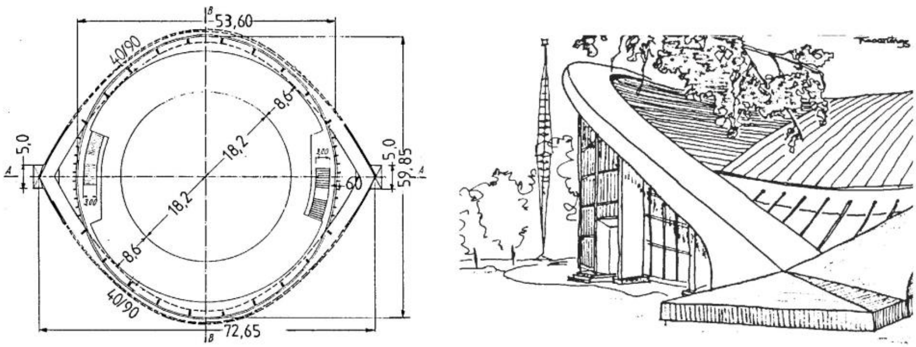

Fig. 5 The Great Hall of the Textile Fair (Source: http://rc5.gaf.ni.ac.rs/dec/arhcons/doc/homes/kostic/Osnovne%20Studije%20Arhitekture/Konstruktivni%20sistemi%20II/07-Viseci-03_KS2.pdf, Accessed: 2017-03-24)

The contour of the building's floor plan s a curve of unknown geometric properties, and it is to be approximated by the proposed procedure as accurately as possible. Its shape and dimensions are taken from the drawings presented in Fig. 5 – left.

The floor plan contour curve is defined by a discrete set of points $\boldsymbol{P}$ of 164 manually selected points $P\,(X_P, Y_P)$, in the same way as it is done in the case of the control model (see Fig. 6 – Approximation I). The points are positioned within the local Descartes coordinate system (of the axes coinciding with the axes of Fair hall ground-base's symmetry).

Numerical values for the boundary conditions X_A and Y_B are $X_A = 36.325\ m$, $Y_B = 31.425\ m$.

The approximation of the floor plan contour by the Weber's curve – parabola (Approximation I) also gives large value for the coefficient of determination: $_IR^2 = 0.998$. However, the maximal value of the Δ_y - *fitting error* is significantly larger: $_I\Delta_y = 0.851\ m$ and thus it is out of the range of the permissive tolerance for the architectural practice. From the latter it can be concluded that the ground-base contour of the object cannot be considered as a parabola!

In accordance with the previous, the following approximation of the contour curve (Approximation II) is performed by the use of a class of the Weber's focal-directorial curves initially supposed to be adequate from their morphological aspects.

The maximal value of the Δ_y - *fitting error*, obtained through the approximation of the ground-base contour with a parabola (${}_I\Delta_y = 0.851\ m$), implies the necessity for defining a control point $C\,(X_C, Y_C) \in \boldsymbol{P}$. Numerical values of its coordinates are $X_C = 26.943\ m$ and $Y_C = 14.988\ m$.

After substituting the boundary conditions $X_A = 36.325\ m$, $Y_B = 31.425\ m$ into the system of equations (8), the following values for the Weber's curve $W_{fd}^{[1,1][1,1,k]}(0)$ parameters are obtained $f = 23.496\ m$, $k = 0.753$, $Y_C = -43.262\ m$.

The selected set of points $\boldsymbol{P}$ on the ground-base contour of the object through the Approximation II (see Fig. 6 – Approximation II), gives the following coefficient of determination ${}_{II}R^2 = 0.9992$. While the numerical value for ${}_{II}\Delta_y$ - *fitting error* of this approximation is ${}_{II}\Delta_y = 0.397\ m$.

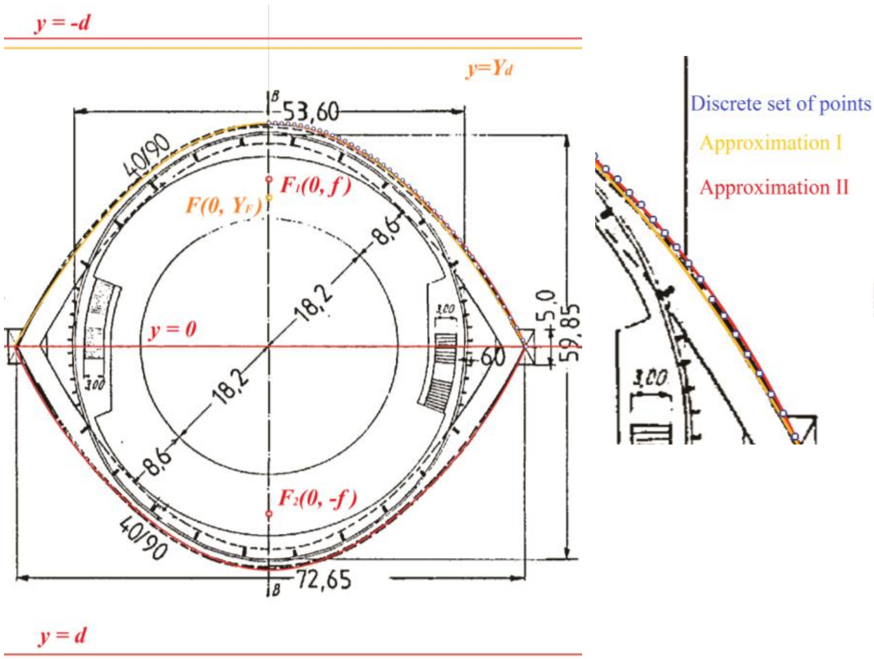

Fig. 6 The contour of the object's ground base The Great Hall of the Textile Fair approximated by parabola and Weber's curve with two foci and three directrices

For architectural practice, the obtained value for ${}_{II}\Delta_y$ - *fitting error* is within permissive tolerance.

Both qualifiers analyzed in the previous section applied to the building's,point out the fact that the approximation by the use of Weber's curves of multiple foci and directrices is a significant improvement, since: ${}_{II}R^2 > {}_IR^2$ and ${}_{II}\Delta_y < {}_I\Delta_y$.

4. CONCLUSION

The diversity of shapes that the Weber's focal-directorial curves offer, provides an opportunity of a proper approximation of many other curves even of those that are set of points found on various curved forms present in an architectural-urban design. Thus, the historically important even ruined buildings can successfully be modelled using computer aided design resulting in an important data base for any further use, such as preservation of historical heritage.

In this paper a particular class of Weber's multifocal and multi directorial plane curves was used in creation of a procedure for an adequate approximation of a ground-base contour line of a Fair Hall building situated a town of Leskovac in Serbia.

In our further research, other classes of Weber's curves both plane and special will be of a particular interest in seeking a solution for optimal approximation of other curves.

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REFERENCES

1. B. Banjac, B. Malešević, M. Petrović and M. Obradović, "A Computer Verification of a Conjecture About The Erdös-Mordell Curve", Proceedings of 21th Telecommunications Forum TELFOR 2013, Belgrade, Serbia, pp. 1031-1034, 2013.
2. B.Banjac, M. Petrović and B. Malešević, "Visualization of Weber's curves and surfaces with applications in some optimization problems", Proceedings of 22nd Telecommunications Forum TELFOR 2014, Belgrade, Serbia, pp. 1031-1034, 2014.
3. B. Conde, L. Díaz-Vilariño, S. Lagüela and P. Arias, "Structural analysis of Monforte de Lemos masonry arch bridge considering the influence of the geometry of the arches and fill material on the collapse load estimation", Construction and Building Materials, Vol. 120, pp. 630-642, 2016.
4. S. Duvernoy and P.L. Rosin, "The Compass, the Ruler and the Computer: An Analysis of the Design of the Amphitheatre of Pompeii", In: Ostwald M. (eds) Architecture and Mathematics from Antiquity to the Future. Birkhäuser, Cham, pp. 525-540, 2015. First published as: Sylvie Duvernoy and Paul Rosin, "The compass, the ruler and the computer", pp. 21–34 in Nexus VI: Architecture and Mathematics, Sylvie Duvernoy and Orietta Pedemonte, eds. Torino: Kim Williams Books, 2006.
5. J.L. Ginovart, J.M.T. Domingo, G.F. Anguera, A.C. Jover and S.P. de Sola-Morales Serra, "The Ellipse and the Oval in the Design of Spanish Military Defence in the Eighteenth Century", Nexus Network Journal, Architecture Mathematics, Vol. 16, pp. 587–612, 2014.
6. J.L. Ginovart, J.M.T. Domingo, A.C. Jover and S.C. Pla, "Quaestiones geometriae in the Amphitheatre of Tarragona", Journal of Cultural Heritage, Vol. 18, pp. 333–341, 2016.
7. J.L. Ginovart, A.C. Jover, S.C. Pla and M.L. Piquer, „Layout of Catenary Arches in the Spanish Enlightenment and Modernism", Nexus Network Journal, Architecture Mathematics, Vol. 19, issue 1, pp. 85–99, 2017.
8. G. Gonzalez, A. Samper and B. Herrera, „Classification by Type of the Arches in Gaudı´s Palau Guell", Nexus Network Journal Architecture and Mathematics, pp 1–14, 2017.
9. S. Huerta, "Structural Design in the Work of Gaudí", Architectural Science Review, Vol. 49/4, pp. 324-339, 2006.
10. D. Huylebrouck, "Curve Fitting in Architecture," Nexus Network Journal, Architecture Mathematics, Vol. 9, No.1, pp. 59–70, 2007.
11. J.S. Dorton Arena, http://www.remhaus.pl/pol_pl_staw_areny.htm, Accessed: 2015-3-25
12. D. Kostić, Contribution to the solution of the stability problem of the double layered catenary systems, Doctoral dissertation (in Serbian: Prilog rešenju problema stabilnosti dvopojasnih lančanih sistema), University of Niš, Faculty of Civil Engineering and Architecture, 2007

13. D. Kostić, Konstruktivni sistemi 2, Viseći sistemi, 2011, available at: http://rc5.gaf.ni.ac.rs/dec/arhcons/doc/homes/kostic/Osnovne%20Studije%20Arhitekture/Konstruktivni%20sistemi%20II/07-Viseci-03_KS2.pdf, Accessed: 2017-03-24
14. N.C. Makoond, “Advanced Computational Modelling of Taq-Kisra”, Msc dissertation, Czech Technical University in Prague, 2015, available at: http://www.msc-sahc.org/upload/docs/new.docs /2015_NChandra.pdf, Accessed: 2017-03-24
15. B. Malešević, M. Petrović, M. Obradović and B. Popkonstantinović “On the Extension of the Erdös-Mordell Type Inequalities”, Mathematical Inequalities and Applications, 17 (1), pp. 269–281, 2014.
16. NCS Libraries, available at: https://d.lib.ncsu.edu/collections/catalog?f%5Bwork_facet%5D%5B%5D=Dorton+Arena+(Raleigh%2C+N.C.)&per_page=50, Accessed: 2017-03-18
17. M. Petrović, M. Obradović and R. Mijailović, “Suitability analysis of Hügelschäffer’s egg curve application in architectural structures’ geometry”, Proceedings of ICEGD Conference, Iasi Romania, Buletinul Institutului Politehnic din Iasi, Publicat de Universitatea Tehnica “Gheorghe Asachi“ din Iasi, Tomul LVII (LXI), Fasc. 3, pp. 115-122, 2011.
18. M. Petrović, B. Banjac and B. Malešević, “The Geometry of Trifocal Curves with Applications in Architecture”, Urban and Spatial Planning SPATIUM International Review, No. 32, pp. 28–33, 2014.
19. M. Petrović, B. Malešević and B. Banjac, “Some surfaces of second order as examples of Weber’s surfaces”, Journal of Industrial Design and Engineering Graphics, Vol. 10, pp. 55–60, 2015.
20. M. Petrović, Generating the focal-directorial geometric forms as a designing pattern of the architectural-urban space (in Serbian: Generisanje fokalno-direktrisnih geometrijskih formi kao obrasca za oblikovanje arhitektonsko-urbanističkog prostora), Doctoral dissertation, University of Belgrade, Faculty of Architecture, 2016, available at: http://www.arh.bg.ac.rs/wp-content/uploads/Doktorati/2016/2016_08_Maja_Petrovic_disertacija.pdf?pismo=lat
21. M. Petrović, B. Banjac, B. Malešević and R. Mijailović, “Curve fitting by multifocal ellipses in Architectural Structures Geometry, Proceedings of 5th International Conference for Geometry and Graphics moNGeometrija 2016, Belgrade, Serbia, pp. 160-164, 2016.
22. A. Samper, G. González and B. Herrera, “Determination of the geometric shape which best fits an architectural arch within each of the conical curve types and hyperbolic-cosine curve types: The case of Palau Güell by Antoni Gaudí”, Journal of Cultural Heritage Vol. 25, pp. 56-64, 2017.
23. Lj. Velimirović, G. Radivojević, M. Stanković and D. Kostić, "Minimal Surfaces for Architectural Constructions", Facta Universitatis: Architecture and Civil Engineering, Vol. 6, No. 1, pp. 89-96, 2008.

PRIMENA VEBEROVIH FOKALNO-DIREKTRISNIH RAVANSKIH KRIVIH U APROKSIMACIJI KONTURNE KRIVE OSNOVE ARHITEKTONSKIH OBJEKATA

Jedan od glavnih ciljeva istraživanja nekih problema u arhitektonskom dizajniranju zgrada je potpuno razumevanje i adekvatno primenjivanje naučnih načela koje arhitekte koriste u svojim projektima. U ovom radu analiziramo mogućnosti primene Veberovih fokalno-direktrisnih krivih u aproksimaciji konture osnove arhitektonskih objekata, tj. zgrada. U vezi sa tim, definisana je Veberova kriva sa m fokusa i n direktrisa. Osim toga, uvedeni su posebni kvalifikatori kako bi se procenio nivo preciznosti izvršene aproksimacije. Važnost istraživanja se posebno iskazuje u činjenici da je postupak kreiran sa mogućnošću primene u arhitektonsko-urbanističkom dizajniranju savremenih oblika, kao i u domenu zaštite i revitalizacije istorijskog nasleđa u smislu stvaranja odgovarajućih geometrijskih modela za dalju upotrebu pomoću računara.

Ključne reči: *Veberove krive, parabola, arhitektonski objekti, aproksimacija*