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ADVANCED DEVELOPMENT OF SPACE STRUCTURES IN DOMAINS OF 3D TRANSFORMATION

A B S T R A C T

The emergence of the new architectural solutions and structural forms of Mengeringhausen, Tsuboi, Safdie, Foster, Calatrava and other creators of magnificent structures, may be taken as an initiation and explosion of inventiveness which has continued up to till present. Consequently, the topic of this paper is to show a part of broad range of structural systems which have not been sufficiently disclosed in Serbia and surroundings, in spite of their attractiveness in contemporary architecture, in terms of space transformations, materialization and technology.

The basic properties of all analyzed space structures lies in their geometric shape (Archimedean and Platonic polyhedra, polyhedron structures, and bionic of structures as well), which applies regularity, symmetry, speed of mounting, as well as modularity of the original matrices. Solutions and analyses shown deal with multifunctional space matrices, which make its potential very important both in architectural design and in structural theory. The topic of this paper is to consider the development of matrix structure in context of architectural forms in future, emphasizing the importance of structural geometry and its possible applications.

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INTRODUCTION

What lies beneath any good architectural design is a constant quest for some kind of order, harmony and balance, for an order of things established in the right way. This order of things, hierarchy and harmony originate from geometry. The early beginnings of geometry are associated with the ancient Greeks, particularly with the theories of Pythagoras and Euclid that differed in understanding of the systems of geometry, but both attributed a mythical context to the geometry itself. This context and a number of analogies emerging from it, the hidden connection between numbers and shapes (Ghyka, Pascal), the basic archetypes of structure and aesthetics of the ancient architecture represent exploratory model for redefined hypothesis about universal structure (and application) of polyhedron architecture.

Among professionals there is a common division into engineering and architecture that represents a simplified thesis – engineering is observed as quantity while architecture is viewed as quality. This quality is visual, but it is not only that. In the history of architecture and construction, structural achievements were in perfect harmony with architectural ones, making an everlasting link between these two complexes: unity and functionality. Once new building technologies, functions and geometries appeared, the contemporary architecture provided the new modalities for creative shaping of architectural solutions and programs. In the context of forming the modern space structures these modalities are based on geometric systems and described by means of entities defined as polyhedrons.

Polyhedrons and polyhedron geometries are theoretically based in the ancient world, whilst Buckminster Fuller invented how to technically apply them in the 1950s [7]. We are talking about cell unit structures that allow creative and almost unlimited possibilities for combining the architectural shapes by creating space frames which define the architectural spaces. New tendencies emerged in the context of architecture redefining the old concept of unity of art and technique as well as associating ethical, esthetical and logical principles with experience and the advantages of techniques (technology) by integrating the artistic act with the new trends in technology.

Just like experimenting and searching for form is one of the unconscious tendencies for overcoming the instabilities, thus generating of polyhedron structure should be understood as creating the systematic balance aiming at overcoming any limitations. These structures may be understood universally as a principle of shaping, defining and solving the problems, classic and modern ones at the same time. Polyhedron geometries have numerous modalities, its systems and possibilities, and they represent universally applicable structures through their geodesic domes, spatially foldable structures, free and amorphous surfaces that are approximated by polyhedron geometries. Analogies with sphere, principles of bionics, analogies with structures of molecular grids, analogies with Golden Section, Fibonacci sequence, Pascal's triangle and the miraculous world connecting and interpreting numbers, shapes and complex systems are the essence of structural system and analogies of interaction in architectural nature. Generally and essentially all these concepts represent one code and a coherent system: multi-functionality and universality. The essence of the problem lies in the concept that one rather elaborate and ambiguously structured system (polyhedron structures), with its numerous geometrical, esthetic, bionic and structural analogies, offers great possibilities for creative architectural and structural design, in the sphere of improvement of functions (spaces), constructions (structures) and form (shapes).

Plato' s approach, Archimedes

Plato (427-347 B.C.) was the first to describe the five regular polyhedra (Fig. 1). In his dialogue *Timaeus* he argues how, out of two kinds of triangles, four polyhedra are formed, and a fifth is mentioned [1,15]. In the same way the Archimedean solids can be formed, but for this two or three kinds of polygons are used. If this is imaginatively created, then all kinds of convex, regular and semi-regular polyhedra appeared. Presenting the exterior of the object is perceived by the observers as convex.

Consequently, according to the spatial standpoint of the observer, polyhedra could be experienced as convex spatial solids as well as concave ones. The later perception demands a higher degree of imaginative involvement of the observers. For the semi-regular solids a similar situation applies. The Archimedean solids (Fig. 2) are easy to construct by the polygons. The polar-Archimedean solids could not be constructed in this way, without the exact knowledge of the respective lengths of the edges and relationships of the angles of the surfaces. An exact construction of the polar-Arhimedean solids (Fig. 3) has to be acquired along with the concept of polarity. In this way Eugene Charles Catalan (1814-94) described them completely for the first time.

Two things could be recognized in both examples of "concave" and "polar": they demand the ability of spatial imagination much more strongly than the

Figure 1 From triangles, into polygons, into polyhedra; after Plato (*Timaeus*)

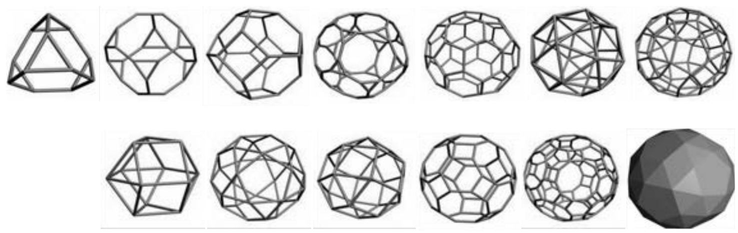

Figure 2 Archimedean solids

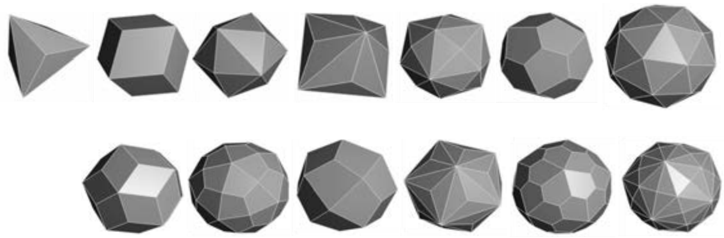

Figure 3 Polar-Archimedean or Catalanian solids

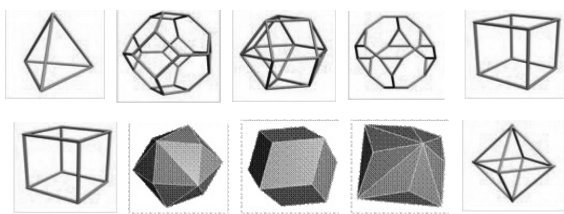

Figure 4 Buckling and depressing regular polyhedra (hexahedron and octahedron)

descriptions of either Plato or Archimedes (c.287 to 212 B.C.). This corresponds to the thesis presented at the outset associated with the development of the ability to conceive and think. On the other hand, the contrary standpoints of the two examples show polar relationships. The convex and concave conceptions face each other as earlier and later discoveries, in the same way as the discoveries of the Archimedean and polar – Archimedean were discovered earlier and later [14].

Metamorphoses – Polar levels and their transitions

These relationships turn stage by stage into their opposite when a regular polyhedron (Fig. 4), through pressing the corners and buckling the surfaces, step by step is changed via Archimedean (or polar-Archimedean) solids into its polar-opposite solid.

Dynamic metamorphoses between poles

Every regular, even semi-regular solid could be changed into its polar opposite. Firstly, a simple observation would be when a force moves towards a point, where the point offers no resistance, the force continues in the same direction beyond the point, leaving it as soon as it has reached it (Fig. 5).

In a similar manner the observer could have impression that the surfaces of any polyhedron are formed by forces which are effective from outside, and the corners as the counter-force effective from inside. If the force increases equally from all sides, the polyhedron becomes smaller and finally is reduced to the point. If the forces continue undiminished in the same direction, instead of forming surfaces from the outside they form corners from the inside. A polihedron becomes the polar opposite of the initial polyhedron.

Figure 5
A force of direction passing through a point

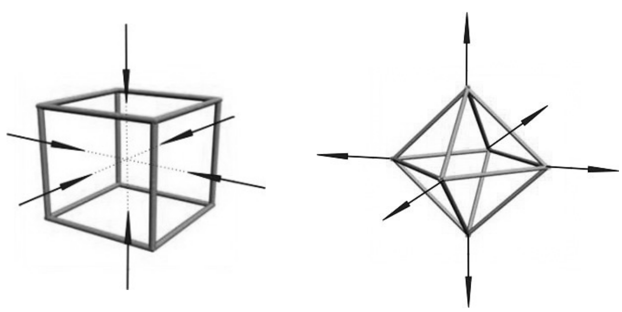

Figure 6
Metamorphosis of directions of force in going through a point (hexahedron and octahedron)

The important thing with this example is the change of one thing into the other. This comes because of the dynamic of the whole process going through the zero-point. The point here marks an essential turning point of metamorphosis supported by the concept of infinity in its smallest form (Fig. 6).

3D TRANSFORMATIONS OF THE SPACE STRUCTURE SYSTEM

Cell structures modalities - using HP shell surfaces

Elements of shells originated from constructive transformations of side supporting arches, placed in slant planes and interconnected by cable construction represent basic cell type. The elements of the objects are analyzed in terms of geometry of large span structures. Thus the analytical approach is generated from basic to complex model. Briefly: basic cell form is made of three identical elements in a shape of hyperbolic paraboloid (HP), then a dual form is made of four identical elements in a shape of HP (and a fifth which represents an inter-element, Fig. 7 and 8) and a triple form which is further reduced to typeset HP elements forming object on large span structures (polyvalent spaces, halls, market halls, etc.).

Diameter of basic cell form (typeset segment) is 60 meters long, which points to the fact that arched elements of hyperbolic paraboloid are also an important dimension that are interconnected by the main ring and within HP they are connected by cables prestressed in both directions. Typeset concrete peels are inserted according to form and shape into suitable places within the network of cables, then covered by mesh reinforcement and finally sprayed by three-centimeter thick concrete layer. Roof is covered with protective foil. In the system of nontransparent hyperbolic paraboloids, there are transparent HP's made of same load-bearing structural elements with aluminum sub-constructions and covers made of glass. Drainage of buildings is designed within the system of supporting elements of HP's.

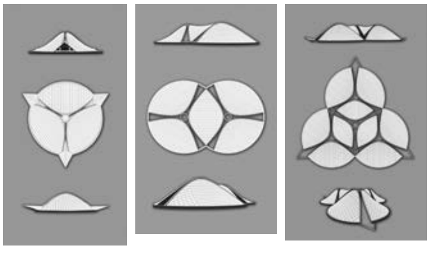

Figure 7 Roof plan and building elevation

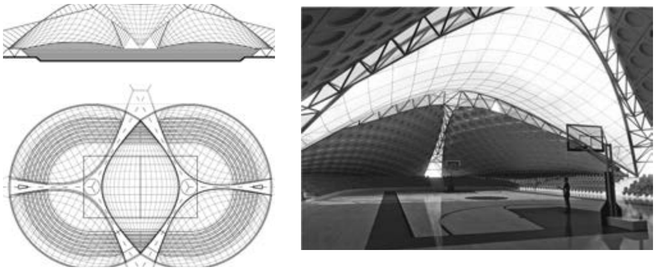

Figure 8 Floor plan and cross-section of the building (multifunctional hall)

Polyhedral structures modalities - using HP folded surfaces

Modalities of polyhedron geometry are applied in the process of forming the residential structures. Typeset elements of Archimedean polyhedra such as truncated tetrahedron, cuboctahedron and truncated octahedron are frequently used (Fig. 9). Elements allow for diverse space transformations: quadrate forms (at the top), triangular (in the middle) and hexagonal structure (at the bottom). Residential units are positioned in primary forms (truncated octahedron) while communications, installations and windows are placed as interconnected structural forms [2,4,6].

Typical rhombic dodecahedron element (dual cuboctahedron) positioned vertically is covered with HP folded surfaces, giving stiffness to the structure but also giving astonishing visual effect. Besides HP's, it is possible to additionally strengthen the construction with folded structures (triangulation) i.e. by combination of the previously described elements. Three-dimensional space frame: dual of polyhedron cuboctahedron represent a basic concept-archetype (Fig. 10).

The example shows modeling possibilities where HP's may occupy different positions in relation to axis and be positioned according to functional demands of inner spaces. Basic purpose of HP elements is stiffening but also creating visual effect (transparent-nontransparent, intensity of daylight in segments, light moods in the interior, reduction of HP elements according to requirements and needs, etc. (Fig. 11).

Figures on the right shows the basic concept of space composition from basic forms-truncated cubes positioned vertically (diagonally). Different options of space integration: frame structures (polyhedra) and three-dimensional space structural systems. Concept of joints presented here allows different possibilities for combining: in respect to both urban and typical housing units that are shown modalities of the basic Archimedean polyhedron (cuboctahedron) in the form of dual (Catalan polyhedra) positioned vertically (Fig. 12). This kind of structure shows good space by functional and structural characteristics - space volume, horizontal and vertical integration [8].

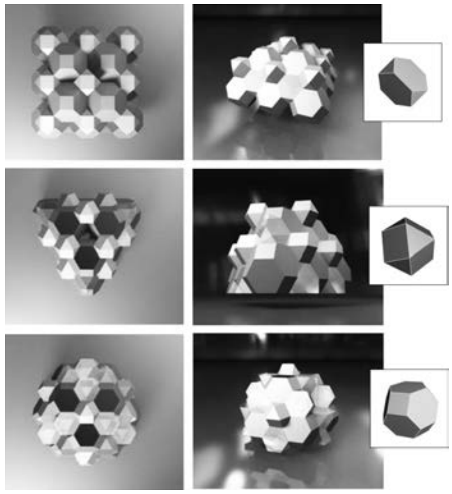

Figure 9 Modalities of polyhedron geometry are applied in the process of forming the residential structures

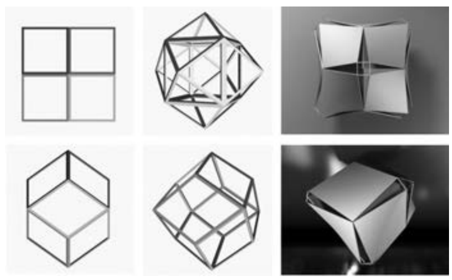

Figure 10 Three-dimensional space frame: dual of polyhedron cubocta-hedron, 12 equal sides and 24 equal edges joint at an angle of 120 degrees

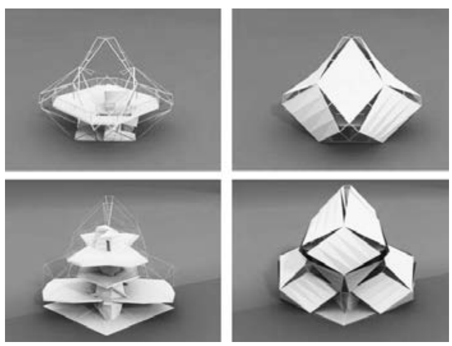

Figure 11 Modalities of the existing structure of three-dimensional frames and HP elements

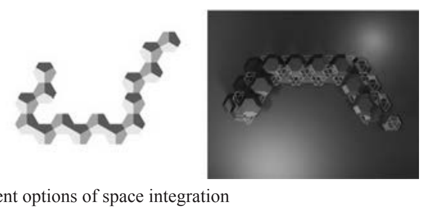

Figure 12 Different options of space integration

Polyhedral structures modalities - using three-dimensional strut elements

Grid geometry analysis

In the next section some basic types of grid-generated square pyramid, as the basic building element are presented. The structures were shown in a hierarchy, so to follow, i.e. emerging one from another, multiplied and improved [10,11] (Fig. 13). Certain structure properties of specific elements that make them (struts, joints...) are briefly reviewed, and, very importantly, total number of standard elements included in each of the structures is shown here (Fig. 14).

The structure hierarchy is designed to expose an evolutionary path formation of a structure rotating hyperbolic cylinders as the final modalities of this analysis [16,17].

Space frame in the plane - the girder is equivalent to the plate, which may be square, rectangular, irregular or polygonal shaped (Fig. 15). There are only few different struts and joints, and from that point of view this makes one of the most economical and mostly applied spatial grid. Basic structure spatial grid unit - flat shaped plate is a regular square pyramid. Basis of the pyramid is quadrilateral in shape. Pyramid height is half of the diagonal of the basis, on which it provides a 45-degree angles at the nodes, and this results in more even distribution of stresses in the struts [5].

Struts of the upper and lower bands are the same in length “a” or “a”, while their profile of the position vary in the grid, and position relative to the supports spatial grid. Connection struts – diagonals all have equal length “c”, and profiles in upper and lower bands vary depending on the position within the grid, in relation to the supports. Space grid is semi-cylinder in geometric shape, which could be of different types and relationships of geometry (curvature, range, size, ...), (Fig. 16). It is derived from the plane grid by shortening the bottom band rods in one direction, which deforms the lower band and the squares are lost, so the rectangular space grid is formed in this manner. The greater the shortening of the bottom band rods, the greater the curvature of the grid, i.e. the greater the arrow of the grid. Economical aspect of this grid (geometry) is still at a high level as compared to the planar grid where only just one type of rod (short rod of the bottom band) appears.

Basic structure unit of the space grid cylinder shape is a regular square pyramid. Basis of the pyramid is quadrilateral in shape. Height of the pyramid is equal to

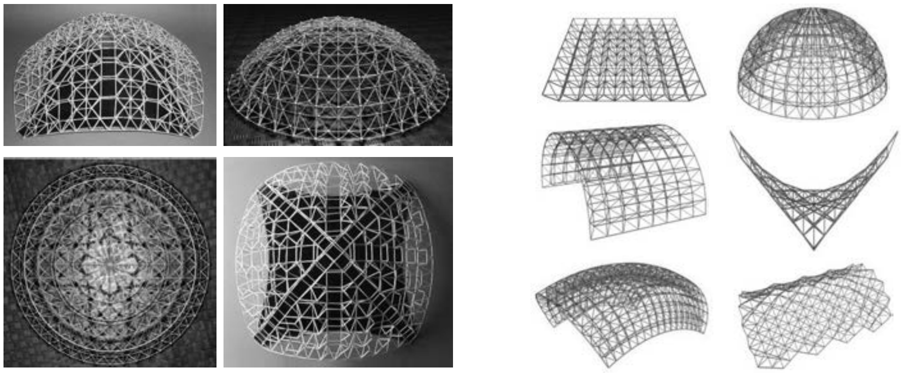

Figure 13 The basic types of grid generated by square pyramid

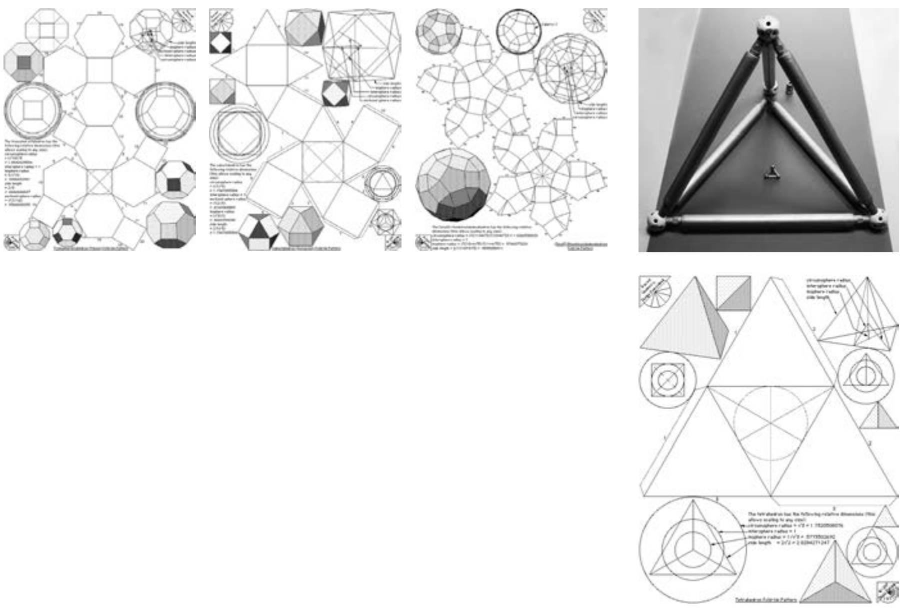

Figure 14 Grid and polyhedral construction elements of three-dimensional structure

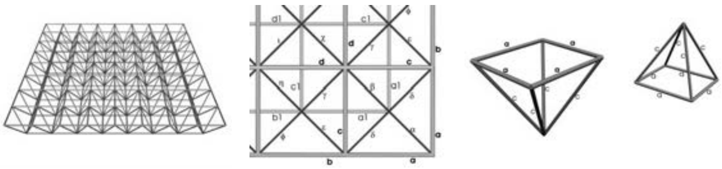

Figure 15 Geometrical space grid of flat shaped plate

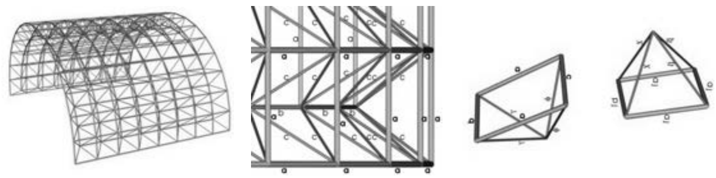

Figure 16 Space grid semi-cylinder geometrical shaped

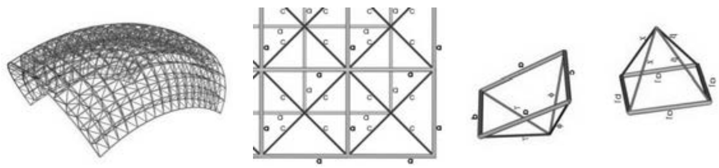

Figure 17 Space grid geometric shape of the torus

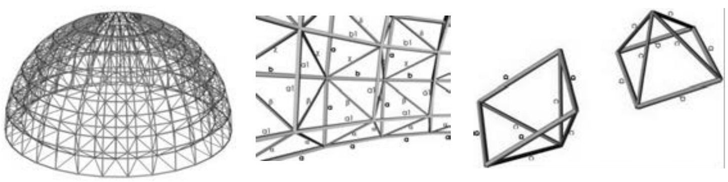

Figure 18 Space grid geometric shape of the calotte

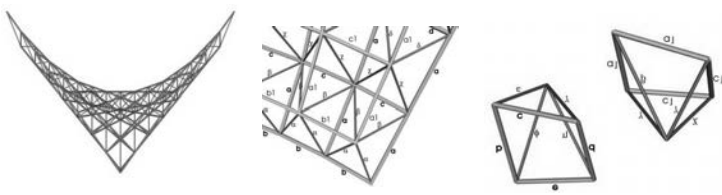

Figure 19 Space grid of hyperbolic paraboloid geometric form

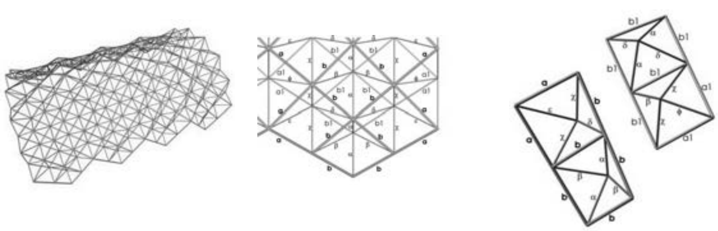

Figure 20 Space grid with geometric shape of rotating hyperbolic cylinder

half of the diagonal of the basis which facilitates 45-degree angles at the nodes, and this results in more even stress distribution in the rods.

The lower rods band “a”, and half of the bottom rods band “a” are the same length, while the other half bottom rods band have length “b”. Profiles depend on the position in the grid, and position relative to the supports of the spatial grid.

Diagonals are all mutually equal - length “c”, and profiles, both at the upper and lower bands, vary depending on the position in the grid and in relation to the supports.

Space grid geometric shape of the torus, which can be of different types and relationships of geometry (curvature in one direction and in second direction, their attitude, range, length, ...), are derived from cylindrical rods by shortening the bottom rods band in the other direction, which deforms the lower band and gets a double curved shape or toroidal grid (Fig. 17). The ratio of the rods shortening in both directions determines the ratio of toroidal radius (small and large). The bottom and lower band are analogous to the geometry, only their dimensions differ. Economical aspect of this grid (geometry) is at a lower level, but there are typed struts and joints.

Basic structure unit of spatial grid shaped torus is a squared pyramid. Basis of the pyramid is a quadrilateral shape (isosceles trapeze). Height of the pyramid is approximately equal to half the diagonal of the basis which enables the angle of about 45 degrees in the nodes, and this results in more even distribution of stresses in the sticks.

The structure is double-curved in the direction of greater curvature having the same struts in the upper band “a” in the lower zone “a_1”. In the direction of smaller curvature there are as many different types of rods, as there are segments “b”, “c”, “d” “e “,...”b1”, “c1”, “d1”, “e1”. Profiles of the struts depend on the position in the grid, and position relative to the spatial grid supports.

Diagonal in the direction of smaller curvature is the same by segments and there are as many types as there are segments in the grid in the direction of greater curvature, “α", "β", "γ", "δ", "ε", "ϕ" while their profiles differ depending on the position in the grid in relation to the supports.

Space grid geometric shape of the calotte, or part of the calotte, which could be of different shapes (different slots, radius, height of grid, ...), (Fig. 18) is derived from the toroidal grid by such type of shortening of the lower band, giving equal radius of curvature in both direction, thus the upper and lower band are deformed in that way so the grid gets calotte shape. The upper and lower bands are analogous to the geometry, only their dimensions differ. Economical aspect of this grid (geometry) is analog to economy of toroidal grid, i.e. at the lower level, but there are some typed struts and joints.

Basic structure unit of spatial grid shape of the calotte is a square pyramid. Basis of the pyramid is a quadrilateral shape (isosceles trapeze). Height of the pyramid is approximately equal to half the diagonal of the basis which enables the angle of about 45 degrees in the joints and this results in more even distribution of stresses in the struts.

The structure is double-curved with a single radius of curvature in both directions. Struts of the upper band "a", "b", "c" "d", ..., rods of the lower band ”a_1“, ”b_1“, ”c_1“, ”d_1“, ..., and diagonal "α”, "β", "γ", "δ", "ε", "ϕ", ..., are the same in size at the horizontal segments of the grid. Profiles of the rods depend on the position in the grid, and position relative to the spatial grid supports.

Space grid of hyperbolic paraboloid geometric form, or any part of it, may be different in shapes (different slots, curves, generating networks, ...), (Fig. 19). The upper and lower band are analogous to the geometry and shape of the hyperbolic paraboloid, only differ in the number of fields, i.e. the bottom band has one box less in both directions. Struts are mostly different among each other, but there is a possibility for typing the struts only by the axes of symmetry of the whole structure getting four types of rods. Economical aspect of this grid (geometry) is at a very low level and because of that, it is rather unfavorable for the building.

Basic structure unit of spatial grid shaped hyperbolic paraboloid is a square pyramid. Basis is the pyramid-shaped space (twisted) quadrilateral. Height of the pyramid is approximately equal to half of the secondary diagonal diagonals of the basis, which provides the angles of approximately 45 degrees in the nodes, and this results in more even distribution of stresses in the struts.

The structure is double-curved with a negative Gaussian curve. Struts of the upper band "a", "b", "c", "d ", ..., rods of the lower band "a_1“, ”b_1“, ”c_1“, ”d_1“, ..., and diagonal “α", "β", "γ", "δ", "ε", "ϕ", ..., are of different sizes, i.e.

hyperbolic paraboloid due to twist has lost regularity in the length of the rods (which are all different, i.e. because of the symmetry of the structure there are max 4 struts of the same length) and in the angles, ranging from 45 degrees and down to very shallow, even impossible angles. Profiles of the struts depend on the position in the grid, and on the position relative to the spatial grid supports.

Space grid with geometric shape of rotating hyperbolic cylinder is defined by multiplication of rotational hyperboloid segments which form a cylinder (Fig. 20). The upper and lower bands are analogous to the geometry and shape of the rotational hyperboloid, but differ in dimension and position, that is executed like one over the other by half the angle involving a field grid. Due to the occurrence of rotational hyperboloids, the structure of the upper and lower bands contains only two types of struts, while the inner space consists of 7 types of struts. All the angles are in range of 45 degrees without hard deformations. Economical aspect of this grid (geometry) is at the high level, especially because of attractiveness of the structure which is very favorable for buildings.

Basic structure unit of spatial grid, with rotating hyperbolic cylinder form, is a quadrilateral pyramid. Basis of the pyramid is shaped like squared space rectangle. Height of the pyramid is approximately equal to half of the diagonal of the basis, which provides the angles of approximately 45 degrees in the nodes, and this results in more even distribution of stresses in the struts.

The structure is double-curved with a negative Gaussian curve. Struts of the upper band as “a” and “b”, the bottom band struts are “a_1” and “b_1”, and diagonal rods are “α", "β", "γ", "δ", "ε", "ϕ" and "η". Profiles of struts depend on the position in the grid, and position relative to the spatial grid supports.

Hyperbolic paraboloid shape grid, has an interesting geometry, but it is very unfavorable for the further exploitation because of the huge number of different struts, and thus the nodes. For these reasons as well as its simplification of such grid, and at the same time retaining its quality characteristics (negative Gaussian curve, playful geometry, ..) geometry of the rotational hyperboloid geometry is applied. Thus, the existing quality of hyperbolic paraboloid shape grid is improved and disadvantages are reduced or completely eliminated. Using a series of segments rotational hyperboloid, the resulting grid is close to half-cylinder shape, so it is called a hyperbolic rotational cylinder. This structure has a limited number of different elements, i.e. specifically explored structure has two types of rods in bands and 7 types of diagonal struts. As for the nods, there are only 6 types, three in the upper band and three in the lower band. The structure is easily upgradeable in terms of length (by simple addition of new segments), while the cross terms must increase the range of segments. Segmentation structure facilitates the construction stages, as well as additional system flexibility.

r – radius of the defined (described) circle
φ - central angle of the box of grid
d - the width of the ring (segment) grid

The upper grid band is defined by the geometry of rotational hyperboloid. Defined radius circle, is in fact half of the range that has a structure. Central angle of the grid box "φ" is the size of which is in function of radius and the desired length of rods in the structure, i.e.if the range is larger, central angle is smaller and vice versa, in order to achieve optimum length of the struts. The size that central angle may have are: $\varphi = 360 / (2 + nx4)$ for n = 3,4,5, ... Width of the structure segment is in function of the central angle, and therefore in function of the length of the struts. Segments width is the size by which we have to achieve the angles between the rods of the upper band of the structure closer to the angle of 90°, i.e. the structures boxes are approximate spatial quadrates.

With such a setting the segments width of the structure would be: $d= | 1.3 | \vee | d= | 2.4 | \vee | d= | 3.5 | \vee | d= | 4.6 | \vee d=$ ... and so the pairs of the rods should be at an angle of 90°. That is: $(1, 3') \perp (1', 3)$, $(2, 4') \perp (2', 4)$, $(3, 5') \perp (3', 5)$... i.e. angles in the nodes A1, A2, A3, A4, ... would be equal and all at 90°.

r 1 - radius of the defined circle of bottom struts band
φ - central angle of a one field grid
d - the width of the ring (segment) of grid

Lower grid band is also defined by the geometry of rotational hyperboloids. Radius of the defined circle, the bottom band of the structure is half smaller than the secondary diagonal grid field of the upper band (i.e. smaller for the height of the building blocks unit-pyramid), compared to circle of the defined upper band. This ensures that the diagonal struts are at an angle of about 45° to the upper and lower band. Central angle of the box of bottom band “φ" in the grid is the same as in the upper band, except that the lower band is slipped off and rotated by half the value of the central angle "φ". Such rotation of the lower band compared to the upper one reaches to the bottom band nodes found in the area of the upper band and enables the diagonal form squared pyramid with a hyperbolic basis. Width of the bottom band segments is equal

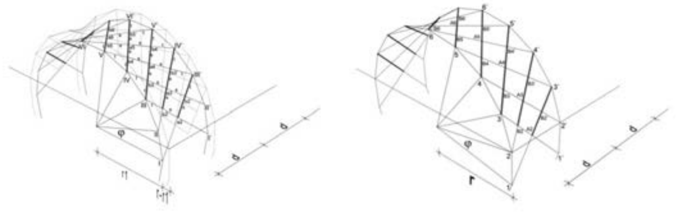

Figure 21 The geometry of the upper and lower struts bend of rotating hyperbolic cylinder

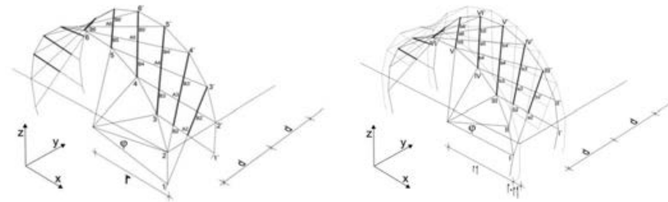

Figure 22 Analytical of the upper and lower band grid

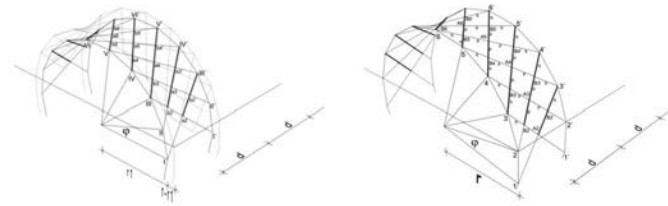

Figure 23 Types of struts of the upper and lower band of rotating hyperbolic cylinder

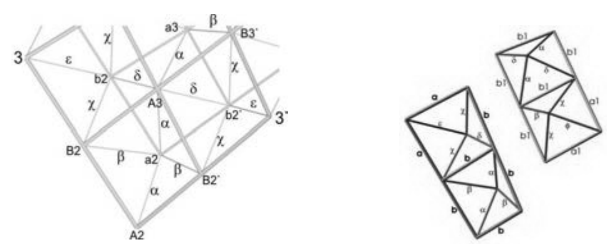

Figure 23a Types of struts – diagonals of rotating hyperbolic cylinder

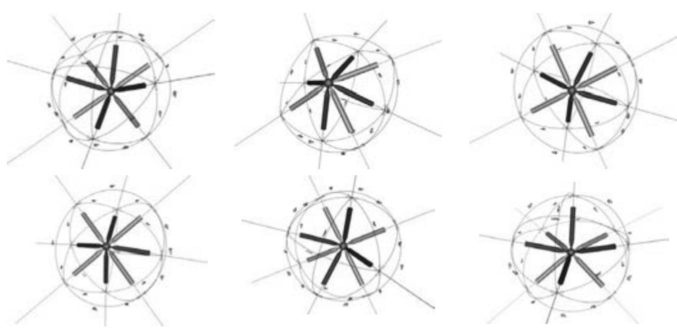

Figure 24 Typed nods 1&4, 5&2 and 3&6

to the width of the upper segment band, thus we get a segment of the structure whose multiplication "m" times gets semi-cylinder derived from rotational hyperboloids (Fig. 21).

r - radius of defined circumcircle
φ - central angle of a field grid
d - the width of the ring (segment) grid

Coordinate system coincides with the defined center circle.

Xn(x,y,z)

Coordinates of nodes*: 1, 2, 3, ...*

n=1, 2, 3, ... n(r×cos[ϕ/2+(n-2)×ϕ], 0, r×sin[ϕ/2+(n-2)×ϕ])

Coordinates of nodes*: 1', 2', 3', ...*

n=1, 2, 3, ... n'(r×cos[ϕ/2+(n-2)×ϕ], m×d, r×sin[ϕ/2+(n-2)×ϕ])

The coordinates of these nodes *1, 2, 3, ...* i *1', 2', 3', ...*, differ only in the "y" coordinates, i.e. determinations after the "x" and "z" coordinates thus changing the "y" coordinates, we can get all the nodes of the type

y = 0, 1 × d, 2 × d, 3 × d, ... m × d;
m = 0, 1, 2, ... number of segment structure

Coordinates of nodes*: A1, A2, A3, ...*

There are several ways to determine these coordinates, and one is that the points *A2, A3, A4,* ... a mid-long | 1 3' |, | 2 4' |, | 3 5' |, ... n = 2, 3, 4, ..., m = 0, 1, 2, ... number of segment structure → An (x, y, z);

An ((x[n-1] + x[n+1]) / 2, d/2 + m×d, (z[n-1] + z[n+1]) / 2)

Coordinates of nodes: *B1, B1', B2, B2', B3, B3', ...*

One way to determine the coordinates of these nodes is that they are the intersections of lines pairs:

B2 = | 2 A3 | ∩ | A2 3 | za n = 2, 3, 4, ...
Bn = | n A(n+1) | ∩ | An n+1 |
Bn' = | n' A(n+1) | ∩ | An (n+1)' |

r1 - radius of defined circumcircle
φ - central angle of a field grid
d - width of the ring (segment) grid

Coordinates of the nodes of the lower band are determined by the same principle as in the upper bend.

Xn(x,y,z)

Coordinates of nodes*: I, II, III, ...*

n = I, II, III, ... n(r1×cos[(n-1)×ϕ], 0, r1×sin[(n-1)×ϕ])

Coordinates of nodes*: I', II', III', ...*

n=I, II, III, ... n'(r1×cos[(n-1)×ϕ], m×d, r1×sin[(n-1)×ϕ])

The coordinates of these nodes *I, II, III*, ... and *I', II', III'*, ... differ only in the "y" coordinate, i.e. after determining the "x" and "z" coordinates thus changing the "y" coordinates, we can get all the nodes of this type: y = 0, 1 × d, 2 × d, 3 × d, ... m × d; m = 0, 1, 2, ... number of segment structure.

Coordinates of nodes*: a1, a2, a3, ...*

There are several ways to determine these coordinates, and one is that the points *a2, a3, a4*, ... mid-and long | I III' |, | II IV' |, | III V' |, ..., n = 2, 3, 4, ..., m = 0, 1, 2, ... number of segment structure → an (x, y, z);

an ((x [n-1] + x [n +1]) / 2, d/2 + m × d, (z [n-1] + z [n +1]) / 2)

Coordinates of nodes*: b1, b1', b2, b2', b3, b3', ...*

One way to determine the coordinates of these nodes is that they are the intersections of lines pairs (Fig. 22):

b2 = | II a3 | ∩ | a2 III | for n=2, 3, 4, ... ∨ n=II,III,IV

bn = | n a(n+1) | ∩ | An (n+1) |

bn' = | n' A(n+1) | ∩ | An (n+1)' |

The upper band structure of the rotating hyperbolic cylinder contains only two types of struts. From the manner of generating the geometry structure (line rotation around the central axis) it ensues that the struts that are on the same "orbit" are of equal length, i.e. that the band has a maximum of 4 types of sticks. However, as the structure is symmetric to the plane perpendicular to the axis of rotation, there remain only two types of struts, and as shown in figure: struts "p" type and struts "q" type (Fig. 23).

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From the previous assumptions about generating the structure of rotational hyperbolic cylinder, and properties that the elements on the same "orbit" are equal, it ensues that the maximum number of different nodes is 5. However, according to symmetry of the structure, this number is reduced to 3 different types of nodes (Fig. 24).

Type 1 _ 3, 4, 5, ..., 3', 4', 5', ...

Nodes 3, 4, 5, ... are on one side of the plane of symmetry, and these nodes are symmetrical to 3', 4', 5' ... It is not a sufficient condition that these nodes are the same. However, each node is symmetrical in relation to the plane which is determined by the rotation axis (central axis for generating structure) and the center of the node. With this condition nodes 3, 4, 5 ... and 3‘, 4’, 5‘ ... are equal.

Type 2 _ B2, B3, B4, ..., B2’, B3’, B4’, ...

For nodes B2, B3, B4, ... and B2‘, B3’, B4‘, ... the same requirement applies as for Type 1. Considering that the condition is accomplished, the nodes are equal.

Type 3 _ A2, A3, A4, ...

Nodes A2, A3, A4, ... are the nodes through which the plane of symmetry of the structure passes, so these nodes are symmetric.

Lower structure band of the rotating hyperbolic cylinder is in complete analogy with the upper band and for it the same conditions and parameters stand. From the manner of generating the structure and properties of symmetry, it ensues that the lower band also has only two types of struts, and as shown in the figure they are: type “s” and type “t” struts (Fig. 23).

The situation is similar with nodes, so again we have only three types of nodes in the band.

Type 4 _ III, IV, V, ..., III’, IV’, V’, ...

Nodes III, IV, V, ... are on one side of the plane of symmetry, and these nodes are symmetrical III’, IV’, V’, ... It is not a sufficient condition that these nodes are the same. However, each node is symmetrical in relation to the plane which is determined by the rotation axis (central axis for generating structure) and the center of the node. With this condition nodes III, IV, V, ... and III‘, IV’, V‘, ... are the same.

Type 5 _ b2, b3, b4, ..., b2’, b3’, b4’, ...

For nodes b2, b3, b4, ... and b2‘, b3’, b4‘, ... the same requirement applies as for Type 4. Considering that the condition is fulfilled, the nodes are equal.

Type 6 _ a2, a3, a4, ...

Nodes a2, a3, a4, ... are the nodes through which the plane of symmetry of the structure passes, and these nodes are symmetric.

Diagonals connect the upper and lower band and they form a squared pyramid with twisted basis.

There are 7 types of diagonal struts in the entire structure, following (Fig. 23a):

| A2 a2 | = | a2 A3 | = | A3 a3 | = | a3 A4 | = α tip
| a2 B2 | = | a2 B2' | = | a3 B3 | = | a3 B3' | = β tip
| B2 b2 | = | b2 B3 | = | B2' b2' | = | b2' B3' |= χ tip
| A3 b2 | = | A3 b2' | = | A4 b3 | = | A4 b3' | = δ tip
| b2 3 | = | b2' 3' | = | b3 4 | = | b3' 4' | = ε tip
| B3 IV | = | B3' IV' | = | B4 V | = | B4' V' | = ϕ tip
| 3 IV | = | 3' IV' | = | IV 4 | = | IV' 4' | = ϒ tip

SAP 2000 NonLinear computer program is used for the study of static characteristics of the rotating hyperbolic cylinder structure. Due to conceptuality and the desire to maintain the focus of the research on geometry of the structure, the test is not done in a classic analysis of the load, but the structure of arbitrarily loaded vertical force of 1KN in each node, and a horizontal force of 1KN in each node. While using such an approximation, we can gain insight into the behavior of "geometry" and its weaknesses and strengths [9].

For easier and better introduction of the characteristics of static structures, three structures of the same type were taken in the analysis, with the same range (20m) and a different arc radius. These three structures are identically loaded, so that their comparative behavior could be observed (Fig. 25).

Complete data obtained from the SAP calculations are not shown (too many data), and in this conceptual research they are not relevant. A couple of characteristic (maximum displacement) data is shown, with the "critical" positions in the structure (Tab.1).

nod 27	UX (mm)	nod 38	UX (mm)	nod 38	UX (mm)
horizontal forces	5.2537	horizontal forces	28.4694	horizontal forces	126.0396
vertical forces	0.2592	vertical forces	-0.2429	vertical forces	1.1905
combination H + V	5.5129	combination H + V	28.2265	combination H + V	127.2301
nod 27	UY (mm)	nod 38	UY (mm)	nod 38	UY (mm)
horizontal forces	-1.0552	horizontal forces	-2.0479	horizontal forces	3.7176
vertical forces	-0.2135	vertical forces	0.0206	vertical forces	0.0463
combination H + V	-1.2687	combination H + V	-2.0274	combination H + V	3.7639
nod 27	UR (mm)	nod 38	UR (mm)	nod 38	UR (mm)
horizontal forces	5.3586	horizontal forces	28.5429	horizontal forces	126.0944
vertical forces	0.3358	vertical forces	0.2437	vertical forces	1.1913
combination H + V	5.6570	combination H + V	28.2992	combination H + V	127.2857

Table 1 Moving nodes in small, medium and large radius arc

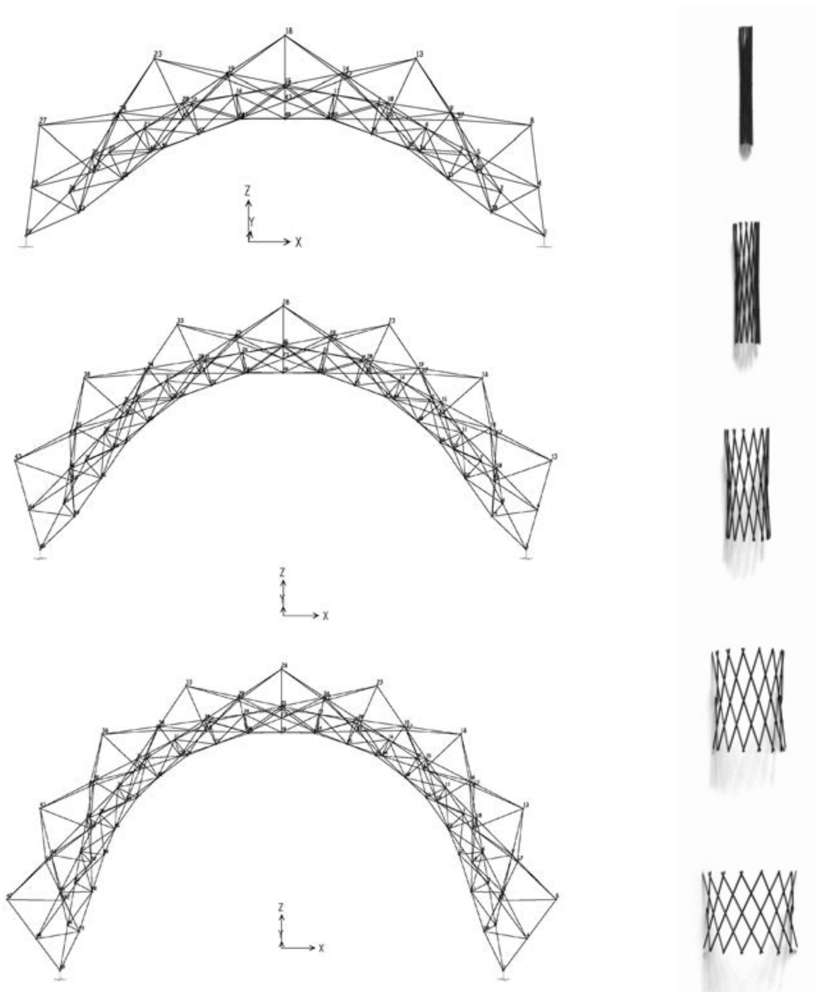

Figure 25 Statical characteristics of the structure

From the data obtained, it is easily deduced that the displacements from the horizontal forces are greater than the displacements due to vertical forces. This would mean that the wind is the dominant influence for designing the structure, and main "problem" that needs to be solved.

The analysis was performed excluding the cover, so that it would be one possible solution of the problem. The cover could assume a stabilizing role, and in that way balance the structure. Such a structure would have a practical cogeneration construction, and very complex effects, and its analysis will not be discussed here.

From a high level of regularity of geometry structure there emerges the possibility for such geometry which could still be exploited. The stretching structure was created as a contribution to this analysis (Fig. 26). In order to make the structure "assembly-removable", further researches go in the direction of applicable systems of connections between struts, and applicable "packages" of struts system. However, in the end, something most logical and simplest turned out, usually not seen at first glance in the structure-geometry: the angles between the two struts change in one plane by reduction in radius of a defined circle [3, 12, 13]. This allows for a very simple connection between the struts, and elegant solution of the problem. The next favorable characteristic of the structure is that with the reduction in radius of a defined circle, despite the change of angles between the struts, the struts retain regularity, i.e. the rods remain even after the "package". Due to the load tendency to "compile", it is necessary to stabilize the structure which remained in the unfolded position and tolerates the load. This could be achieved in several ways: by fixing the supports, by inserting additional rods (struts under pressure).

Another way to stabilize the structure could be via cover. Cover, which would create a "shell" on stretching structure, i.e. solid cover, or soft, membrane cover, which would provide additional reinforcement of the rigidity of the whole structure.

Such stretching structure could be widely applied in temporary prefabricated removable objects. Besides the possibility of removing, replacing and rearranging, it is possible to add and remove the segments, which changes the length of the structure. Such flexibility opens the possibility of use for different purposes.

CONCLUSION

The objective of this paper and the research results is to show a part of modern space structure possibilities defined through geometric patterns (polyhedra and mesh surfaces) and to point to several important features of such systems in space, structural and modular context:

1. Main distinctive feature of polyhedron systems is typeset of the basic element (unit - cell);
2. Elements may be combined and integrated in space without restrictions;
3. Elements have distinctive features of typeset structure grid and other parts of the construction;
4. They are suitable for implementation in different urban contexts;
5. Elements are of prefabricated types, easy to remove, replace and rearrange;
6. They are quickly and easily mounted and suitable for mass production;
7. Elements are surrounding-oriented: multiple orientations, brightness, ventilation, possibility of bioclimatic principles application (solar homes: active and passive);
8. Polyhedron geometries and mesh surfaces allow for great modeling alternatives;
9. Practice shows successful examples of implementation of polyhedron structures.

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NOTES

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